TRB MATHEMATICS

DIFFERENTAL EQUATIONS

Unit-VIII - Differential Equations

Linear differential equation - constant co-efficients - Existence of solutions – Wrongskian - independence of solutions - Initial value problems for second order equations - Integration in series - Bessel's equation - Legendre and Hermite Polynomials - elementary properties - Total differential equations - first order partial differential equation - Charpits method

DIFFERENTIAL EQUATIONS

Definition:

An equation involving one dependent variable and its derivatives with respect to one or more independent variables is called a **Differential Equation**.

Differential equation are of two types.

(i) Ordinary Differential equation

(ii) Partial Differential equation

Ordinary Differential equation

An ordinary differential equation is a differential equation in which a single independent variable enters either explicitly or implicitly.

(i) $\frac{dy}{dx} = x + 5$ (ii) $(y')^2 + (y')^3 + 3y = x^2$ (iii) $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 3y = 0$

Partial Differential equation

A partial differential equation is one ine which at least two independent variable occur.

Example :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$$

Order

The **order** of a differential equation is the order of the highest differential coefficient

degree

The **degree** of the differential equation is the degree of the highest order derivative after removing radicals and fractions

Problems

Find the oder and degree of the differential equation

1. $[1 + \left(\frac{dy}{dx}\right)^2]^3 = (\frac{d^2y}{dx^2})^2$

order = 2 , degree = 2

2. $\cos x\frac{d^2y}{dx^2} + \sin x\left(\frac{dy}{dx}\right)^2 + 8y = \tan x$

order = 2 , degree = 1

3. $\frac{d^2y}{dx^2}+\sqrt{1+\left(\frac{dy}{dx}\right)^3}=0$

order = 2 , degree = 2

4. $\frac{d^2y}{dx^2}=[4+\left(\frac{dy}{dx}\right)^2]^{\frac{3}{4}}$

order = 2 , degree = 4

5. $\frac{d^2y}{dx^2}+x=\sqrt{y+\left(\frac{dy}{dx}\right)}$

order = 1 , degree = 2

6. $(1+y')^2=y'^2$

order = 1 , degree = 1

7. y =4$\frac{dy}{dx}$ +3x$\frac{dx}{dy}$

order = 1 , degree = 2

8. sinx(dx+dy) = cosx(dx-dy)

order = 1 , degree = 1

9. $\frac{d^3y}{dx^3}+(\frac{d^2y}{dx^2})^3+\left(\frac{dy}{dx}\right)^5$+y = 7

order = 3 , degree = 1

10. $\frac{d^4x}{dt^4}+\frac{d^2x}{dt^2}+\left(\frac{dx}{dt}\right)^5=e^t$ **(TRB)**

order = 4 , degree = 1

Formation of differential equations :

Let $f(x, y, c_1) = 0$ be an equation containing x, y and one arbitrary constant c_1. If c_1 is eliminated by differentiating $f(x, y, c_1) = 0$ with respect to the independent variable once, we get a relation involving x, y and $\frac{dy}{dx}$

If we have an equation $f(x, y, c_1, c_2) = 0$ containing two arbitrary constants $c1$ and c_2, then by differentiating this twice, we get three equations (including f). If the two arbitrary constants c_1 and c_2 are eliminated from these equations, we get a differential equation of second order.

Problems

Form the differential equation by eliminating arbitrary constant

1. y = ax+a^2

Differentiating with respect ‘x’

$\frac{dy}{dx}$ = a

differential equation is, y =$\frac{dy}{dx}$ x +$\left(\frac{dx}{dt}\right)^2$

2. y = Acosx+Bsinx

   Differentiating with respect ‘x’

   $\frac{dy}{dx}$ = -Asinx+Bcosx

Again Differentiating with respect ‘x’

   $\frac{d^2y}{dx^2}$ = -Acosx-Bsinx = -(Acosx+Bsinx) =-y

   $\frac{d^2y}{dx^2}$ +y = 0

3. y = Acoskx+Bsinkx

   Differentiating with respect ‘x’

   $\frac{dy}{dx}$ = -Aksinx+Bkcosx

   Again Differentiating with respect ‘x’

   $\frac{d^2y}{dx^2}$ = -Ak^2cosx-Bk^2sinx = -k^2(Acosx+Bsinx) =-k^2y

   $\frac{d^2y}{dx^2}$ +k^2y = 0

4. y = Acos4x+Bsin4x

   Differentiating with respect ‘x’

   $\frac{dy}{dx}$ = -4Asinx+4Bcosx

   Again Differentiating with respect ‘x’

   $\frac{d^2y}{dx^2}$ = -16Acosx-16Bsinx = -16(Acosx+Bsinx) =-y

   $\frac{d^2y}{dx^2}$ +16y = 0

5. y^2=Ax^2+Bx+C

   Differentiating with respect ‘x’

   2yy’ = 2Ax+B

Again Differentiating with respect ‘x’

   2[yy’’+y’y’] = 2A

Again Differentiating with respect ‘x’

yy'''+3y''y'+2y'y'' = 0 ⇒yy'''+3y''y' = 0 (or) $y\frac{d^3y}{dx^3}+3\frac{d^2y}{dx^2}\times\frac{dy}{dx}=0$

6. y=A$e^{3x}+Be^{-3x}$

   y' = $3Ae^{3x}-3Be^{-3x}$

   y'' = $9Ae^{3x}+9Be^{-3x}$ = 9(A$e^{3x}+Be^{-3x}$)

   y'' =9y ⇒ **y''-9y = 0**

7. y=A$e^{3x}+Be^{5x}$ -------------------- (1)

   y' = $3Ae^{3x}+5Be^{5x}$ --------------- (2)

   y'' = $9Ae^{3x}+25Be^{5x}$ ------------(3)

From equation (1),(2) and (3) eliminating A,B

$$\begin{vmatrix} y & 1 & 1 \\ y' & 3 & 5 \\ y'' & 9 & 25 \end{vmatrix} = 0$$

y'' -8y'+15 = 0 **(OR)**

y = A$e^{p1x}+Be^{p2x}$ is a solution of (D-p_1)(D-p_2) y= 0

$(D-3)(D-5)y=0$ ⇒ (D^2-8D+15)y = 0 (or) $\frac{d^2y}{dx^2}$-8$\frac{dy}{dx}$ +15y = 0

8. y = $c_1e^{-x}+c_2e^{2x}$ **(TRB)**

   (D-1)(D+2)y = 0

   (D^2-D+2)y=0 ⇒ $\frac{d^2y}{dx^2}$ - $\frac{dy}{dx}$ +2y = 0

9. y=A$e^{2x}+Be^{-2x}$

   (D+2)(D-2)y = 0

   (D^2 +4)y=0 ⇒ $\boldsymbol{\frac{d^2y}{dx^2}}$ **4y = 0**

10. y = e^{3x}(Ccos2x+Dsin2x)

    ye^{-3x} = Ccos2x+Dsin2x --------- (1)

Differentiating with respect 'x'

y(-3e^{-3x})+ e^{-3x}y' =-2Csin2x+2Dcos2x

e^{-3x} [-3y+y'] = -2Csin2x+2Dcos2x

Again Differentiating with respect 'x'

e^{-3x}(-3y'+y'')-3(3y+y') e^{-3x} = -4Ccos2x-4Dsin2x = -4 (Ccos2x+Dsin2x)

e^{-3x} [y''-6y'+9y] = -4ye^{-3x} ⇒ y''-6y'+13y = 0 (OR)

If $p=\alpha\pm i\beta$,then Differential equation,

(D^2-(sum of roots)D+Product of roots)y = 0

$(D^2-(3+i2+3-i2)D+(3+i2)(3-i2))y = 0$

$(D^2-6D+13)y = 0$

11. $y = (Ax+B)e^{3x}$

Differential equation is, $(D-3)(D-3)\,y = 0$

$(D^2-9)y=0$

Solutions of first order and first degree equations:

(i) Variable separable method

(ii) Homogeneous differential equation

(iii) Linear differential equation

Variable separable :

Variables of a differential equation are to be rearranged in the form

$f_1(x)\, g_2(y)\, dx + f_2(x)\, g_1(y)\, dy = 0$

i.e., the equation can be written as

$f_2(x)g_1(y)dy = -\ f_1(x)\, g_2(y)\, dx$

$\frac{g_1(y)}{g_2(y)} dy = \frac{f_1(x)}{f_2(x)} dx$ ***and integrating on both sides***

$$\int \frac{g_1(y)}{g_2(y)} dy = \int \frac{f_1(x)}{f_2(x)} dx$$

1. $\frac{dy}{dx} + \left(\frac{1-y^2}{1-x^2}\right)^{\frac{1}{2}} = 0$

$$\frac{dy}{\sqrt{1-y^2}} = \frac{dx}{\sqrt{1-x^2}}$$

$$integrating \int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{\sqrt{1-x^2}}$$

$$sin^{-1}y - sin^{-1}x = c$$

2. Solve $\frac{dy}{dx} = y^2x^3$

$$\frac{dy}{y^2} = \frac{dx}{x^3}$$

$$integrating \int \frac{dy}{y^2} = \int x^3 dx$$

$$-\frac{1}{y} = \frac{x^4}{4} + c$$

3. Solve $\frac{dy}{dx} = \frac{y+2}{x-1}$

$\frac{dy}{y+2} = \frac{dx}{x-1}$

$integrating \int \frac{dy}{y+2} = \int \frac{dx}{x-1}$

log(y+2) = log(x-1)+logc

log($\frac{y+2}{x-1}$) $= logc$

$\frac{y+2}{x-1}$ = c

4. Solve e^xtanydx+(1-e^x)$sec^2 ydy = 0$

$\frac{e^x}{1-e^x}$ dx = -$\frac{sec^2 ydy}{tany}$

$integrating \int \frac{e^x}{1-e^x}$ dx $= -\int \frac{sec^2 ydy}{tany}$

-log(1- e^x) = - logtany+logc

log($\frac{tany}{1-e^x}$) = logc

$\frac{tany}{1-e^x}$ = logc

5. Solve x$\sqrt{1+y^2}$ +y$\sqrt{1+x^2}\,\frac{dy}{dx}$

$\frac{xdx}{\sqrt{1+x^2}}$ = - $\frac{ydy}{\sqrt{1+y^2}}$

$integrating \int \frac{xdx}{\sqrt{1+x^2}} = -\int \frac{ydy}{\sqrt{1+y^2}}$

$\times$ 2$\Rightarrow$ $\int \frac{2xdx}{\sqrt{1+x^2}} = -\int \frac{2ydy}{\sqrt{1+y^2}}$

2$\sqrt{1+x^2}$ = -2$\sqrt{1+y^2}$ +c

6. Solve $\sqrt{1-x^2}\ sin^{-1}xdy + ydx = 0$

$\frac{dy}{y} = -\frac{dx}{sin^{-1}x\sqrt{1-x^2}}$

$integrating \int \frac{dy}{y} = -\int \frac{dx}{sin^{-1}x\sqrt{1-x^2}}$

$\int \frac{dy}{y} = -\int \frac{du}{u}$; let u = $sin^{-1}x$,du = $\frac{1}{\sqrt{1-x^2}}dx$

logy = - logu+logc

yu = c $\Rightarrow$ y$sin^{-1}x$ = c

7. Solve ydx-xdy+3$x^2y^2e^{x^3}dx$ = 0

$\frac{ydx - xdy}{y^2}$ +3$x^2e^{x^3}dx$ = 0

d($\frac{x}{y}$) + 3$x^2e^{x^3}dx$ = 0

$integrating \int d(\frac{x}{y}) + \int 3x^2 e^{x^3} dx = 0$

$\frac{x}{y} + e^{x^3} = c$

Homogeneous Differential Equations

A differential equation of the form $\frac{dy}{dx} = \frac{f(x,y)}{g(x,y)}$ OR $\frac{dy}{dx} = f(\frac{y}{x})$ is called a Homogeneous equation.Where f(x,y) and g(x,y) are homogeneous equation of the same degree.

Ex: $\frac{dy}{dx} = \frac{2xy}{x^2+y^2}$

In such case, $\mathbf{y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}}$

1. Solve $\frac{dy}{dx} = \frac{y}{x} + tan\frac{y}{x}$

   Put y = vx $\Rightarrow \mathbf{\frac{dy}{dx} = v + x\frac{dv}{dx}}$

   $\mathbf{v + x\frac{dv}{dx} = v + tanv}$

   $\mathbf{x\frac{dv}{dx} = tanv}$

   $\frac{dv}{tanv} = \frac{dx}{x}$

$integrating \int \frac{dv}{tanv} = \int \frac{dx}{x}$

logsinv = logx+logc

sinv = xc , put v= $\frac{y}{x}$

$\mathbf{sin\frac{y}{x} = xc}$

2. Solve $\frac{dy}{dx} + \frac{y}{x} = \frac{y^2}{x^2}$

   Put y = vx $\Rightarrow \mathbf{\frac{dy}{dx} = v + x\frac{dv}{dx}}$

   $\mathbf{v + x\frac{dv}{dx} + v = v^2}$

   $\mathbf{x\frac{dv}{dx} = v^2 - 2v}$

   $\frac{dv}{v^2-2v} = \frac{dx}{x}$

   $\frac{dv}{v(v-2)} = \frac{dx}{x}$ $\{ \frac{1}{v(v-2)} = \frac{A}{v} + \frac{B}{v-2}$, A = $-\frac{1}{2}$ B = $\frac{1}{2}$

   $-\frac{1}{2}\int \frac{dv}{v} + \frac{1}{2}\int \frac{dv}{v-2} = \int \frac{dx}{x}$

   $-\frac{1}{2}logv + \frac{1}{2}\log(v-2)$ = logx+logc

$\frac{v-2}{v} = x^2c$, put v = $\frac{y}{x}$

v-2 = vx^2c

$\frac{y}{x}$ - 2 = $\frac{y}{x} \times x^2c$

$y - 2x = yx^2c$

3. On putting y = vx, the homogenous differentioal equation $x^2dy + y(x+y)dx = 0$ Becomes xdv+(2v+v^2)$dx = 0$
4. Solve xdy-ydx = $\sqrt{x^2+y^2}$ dx

   Ans: xdv – $\sqrt{1+v^2}$ dx = 0
5. Solve.$\frac{dy}{dx} = \frac{y^3+3x^2y}{x^3+3x\ \ y^2}$

   Ans: $\frac{dx}{x} = \frac{(1+3v^2)}{2v(v^2-1)}$ dv

Linear differential equations

- The linear equqtion in y of the first order is of the form $\frac{dy}{dx}$ +Px = Q(x) , where P , Q are functions in x

  The general solution is y(I.F) = $\int Q\ (I.F)dx + c$

  Where Integral Factor(I.F) = $e^{\int Pdx}$
- The linear equqtion in x of the first order is of the form $\frac{dx}{dy}$ +Py = Q(y) , where P , Q are functions in y

  The general solution is yxI.F) = $\int Q\ (I.F)dy + c$

  Where Integral Factor(I.F) = $e^{\int Pdy}$

Note :

Integral factor of D.E is (I.F)

Then, P = $\frac{(I.F)'}{I.F}$

1. Find the integral factor of (1- x^2)$\frac{dy}{dx}$ +2xy = x$\sqrt{1-x^2}$

   ÷ (1- x^2) ⇒ $\frac{dy}{dx} + \frac{2x}{(1-x^2)}y = \frac{x\sqrt{1-x^2}}{(1-x^2)}$

   P = $\frac{2x}{(1-x^2)}$, Q = $\frac{x\sqrt{1-x^2}}{(1-x^2)}$

   I.F = $e^{\int Pdx} = e^{\int\frac{2x}{(1-x^2)}dx} = e^{-\log(1-x^2)} = \frac{1}{(1-x^2)}$
2. Find I.F of $\frac{dy}{dx}$+ycotx = sin2x

   I.F = $e^{\int Pdx} = e^{\int cotdx} = e^{\log sinx} = sinx$
3. Find I.F of (1+x^2)$\frac{dy}{dx} + y = e^{tan^{-1}x}$

÷ (1-+ x^2) ⇒ $\frac{dy}{dx} + \frac{1}{(1+x^2)} y = \frac{e^{tan^{-1}x}}{(1+-x^2)}$

I.F = $e^{\int \frac{1}{(1+x^2)} dx}$ = $e^{tan^{-1}x}$

4. Find the solution of $\frac{dy}{dx}$ + $\frac{2x}{(1+x^2)} y$ = 0

   I.F = $e^{\int \frac{2x}{(1+x^2)} dx}$ = $e^{\log(1+x^2)}$ = (1+x^2)

   The solution is, y(I.F) = $\int Q\ (I.F)dx + c$

   y(1+x^2) = $\int 0dx + c$

   y(1+x^2) = c

5. If the integral factor of D.E is secx, find P(x)

   P(x) = $\frac{(I.F)'}{I.F}$ = $\frac{secxtanx}{secx}$ = tanx

6. If e^x is an integral factor of D.E . Find P(x)

   P(x) =$\frac{(I.F)'}{I.F}$ = $\frac{e^x}{e^x}$ = 1

7. If $e^{f(x)}$ is IF of D.E , then P(x) = $f'(x)$

Exercise

1. Solve ,$\frac{dy}{dx}$+y = x , Ans: $e^x(y - x + 1) = c$
2. Find IF of $\frac{dy}{dx}$+xy = x , ans: $e^{\frac{x^2}{2}}$
3. Find IF of $\frac{dy}{dx} + \frac{y}{x} = \sin(x^2)$ Ans : x
4. Solve $\frac{dy}{dx}$+2ytanx = sinx , Ans: y = cosx+Ccos^2x
5. Find I.F of $\frac{dy}{dx} - 3y = 6$, Ans: e^{-3x^2}
6. Find the solution of $\frac{dy}{dx} + 2y - 3$ =0 , Ans: y$e^{2x} = \frac{3e^{2x}}{2} + c$

<u>Bernoulli's equation</u>

An equation of the form $\frac{dy}{dx} + py = Qy^n$, Where P,Q are functions in x only.

÷ $y^n \Rightarrow y^{-n}\frac{dy}{dx} + Py^{1-n} = Q$ ----------------------(1)

$Put\ z = y^{1-n}$

$\frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx}$

$\frac{1}{(1-n)}\frac{dz}{dx}$ =$y^{-n}\frac{dy}{dx}$

(1) ⇒ $\frac{1}{(1-n)}\frac{dz}{dx}$ +Pz = Q ,which is linear equation in z

1. Solve , (x+1)$\frac{dy}{dx}$ +1 = $2e^{-y}$

   $\div e^{-y}$ ⇒ $e^{y}(x+1)\frac{dz}{dx} + e^{y} = 2$ ---------- (1)

   $Let\ z = e^{y}$ ⇒ $\frac{dz}{dx} = e^{y}\frac{dy}{dx}$

   (1)⇒ (x+1) $\frac{dz}{dx}$+z = 2

   ÷ (x+1) ⇒ $\frac{dz}{dx}+\frac{1}{(x+1)}z = \frac{2}{(x+1)}$

   $I.F = e^{\int\frac{dx}{(x+1)}=(x+1)}$

   The general solution is, z$e^{\int pdx} = \int Qe^{\int Pdx}\,dx + c$

   Z(x+1) = $\int\frac{2}{(x+1)}$ (x+1) dx +c

   Z(x+1) = 2x +c

2. Find the I.F of $\frac{dy}{dx}(x^2y^3 + xy) = 1$

   $x^2y^3 + xy = \frac{dx}{dy}$

   $\div x^2 \Rightarrow \frac{1}{x^2}\frac{dx}{dy}-\frac{y}{x} = y^3$ -------------------(1)

   $put\ z = -\frac{1}{x}$

   $\frac{dz}{dy} = \frac{1}{x^2}\frac{dx}{dy}$

   (1)⇒ $\frac{dz}{dy}$+zy =y^3

   (2)$I.F = e^{\int pdy} = e^{\int ydy}$=$e^{\frac{y^2}{2}}$

Exercise

1. Find I F of $\frac{dy}{dx} + xtany = x^3$,Ans: $e^{\frac{x^2}{2}}$
2. Find the I.F of $\frac{dy}{dx} + ycosx = y^n$sin2x , Ans: I F = $e^{-(1-n)sinx}$

Equation of the first order with higher degree

We shall denote $\frac{dy}{dx}$ by ‘p’

The D.E of the first order and nth degree, $p^n + A_1p^{n-1} + A_2p^{n-2A}$+.. . +$A_n = 0$

Where $A_1, A_2, A_3, \ldots \;\; ..A_n$ denote functions of x and y

In this case resolve the given equation into factor of first degree,

(p-R_1) (p-R_2) (p-R_3) . . . . .. (p-R_n) = 0

This implies, p =R_1,p =R_2 ,p =R_3,. . . . .. p=R_n

Solve the equation ,then we get,

$\varphi_1(x, y, c_1) = 0$, $\varphi_2(x, y, c_2) = 0$, $\varphi_3(x, y, c_3) = 0$, ……,$\varphi_n(x, y, c_n) = 0$,

Where $c_1, c_2, c_3, \ldots\ldots, c_n$ are constant

The solution of D.E is, $\varphi_1(x, y, c_1)\ \varphi_2(x, y, c_2)\ \varphi_3(x, y, c_3)\ldots\ldots,\varphi_n(x, y, c_n) = 0$

1. Solve, $p^2 - 5p + 6 = 0$ (Or) $(\frac{dy}{dx})^2 - 5\frac{dy}{dx} + 6 = 0$

   ⇒ (p-2)(p-3)= 0

   $p = 2, p = 3$

   $\frac{dy}{dx}$ = 2 , $\frac{dy}{dx}$ = 3

   $dy = 2dx$, dy = 3dx

   $Integrating,$ y = 2x+c_1 ,y = 3x+c_2

   y - 2x- $c_1 = 0$,y - 3x$-c_2 = 0$

   The solution is,(y - 2x- c_1)(y - 3x$-c_2$) = 0

2. Solve, p(p-y) = x(x+y)

   $p^2 - py - x^2 - xy = 0$

   $p^2 - x^2 - y(x + p) = 0$

   (p+x)(p-x) – y(x+p)=0

   (p+x)(p-x-y) = 0

   P = -x , p = x+y

   $\frac{dy}{dx}$ = -x ⇒ y = -$\frac{x^2}{2} + c_1$ ⇒(2y+$x^2 - c_1$) = 0

   $\frac{dy}{dx}$ = x+y ⇒ $\frac{dy}{dx}$ -y= x

   $I.F = e^{\int -dx} = e^{-x}$

   The solution, y$e^{-x} = \int x\, e^{-x} dx + c_2$

   y$e^{-x} = -xe^{-x} - e^{-x} + c_2$ ⇒ (y+x+1 - $c_2 e^{-x}$) = 0

   the solution is, ⇒(2y+$x^2 - c_1$)(y+x+1 - $c_2 e^{-x}$) = 0

3. Solve , $x^2p^2 + 3xyp + 2y^2$ = 0 (TRB)

$x^2p^2 + 2xyp + 3xyp + 2y^2$ = 0
$xp(xp + 2y) + y(xp + 2y)$ = 0
$(px + y)(xp + 2y) = 0$
Px = -y , xp = -2y
$\frac{dy}{dx}$ x = -y ⇒ logy = -logx+logc_1 ⇒ (xy -$c_1) = 0$
xp = -2y ⇒ $\frac{dy}{dx}$ x = -2y ⇒ logy = -2logx+logc_1 ⇒ (yx^2 -$c_2) = 0$
The solution is, (xy -c_1)(yx^2 -c_2) =0

Exercise

4. Solve , $p^2 - 7p + 10 = 0$,Ans: (y - 2x- c_1)(y - 5x$-c_2) = 0$
5. Solve, $p^2 - 7p + 12 = 0$,Ans: (y - 42x- c_1)(y - 3x$-c_2) = 0$]

DIFFERENTIAL EQUATIONS TEST -1

1. What is the order and degree of the differential equation $\frac{d^2y}{dx^2}$+$\sqrt{1 + (\frac{dy}{dx})^2}$=0
   (a) First order, second order (b) second order,first degree
   (c) first order, first degree (d) second order, second degree
2. The differential equation derive from y = A$e^{2x} + Be^{-2x}$ have the order,when A,B are constants
   (a) 3 (b) 2 (c) 1 (d) None of these
3. The differential equation of y = A$e^{3x} + Be^{5x}$ is
   (a) y'' -8y+15y = 0 (b) y'' +8y+15y = 0 (c) y'' +8y = 0 (d) y'' = 0
4. The differential equation of x = Acos(pt-α) is
   (a) $\frac{d^2x}{dt^2}$ =0 (b) $\frac{d^2x}{dt^2}$ =-p^2x (c) $\frac{d^2x}{dt^2}$ =0 (d) $\frac{dx}{dt}$ =-px
5. The solution of $\frac{dy}{dx}$ = 2xy is
   (a) c= logy+x (b) c =logy-x^2 (c) c =xy (d) c = ylogx
6. The degree of the differential equation of $(\frac{d^2y}{dx^2})^3 + (\frac{dy}{dx})^2 + sinx + 1 = 0\ is$
   (a) 1 (b) 2 (c) 3 (d) 4
7. The number of arbitrary constants in the general solution of differential equation of fouth order is
   (a) 1 (b)2 (c) 3 (d) 4

8. A homogeneous differential equation of the form $\frac{dx}{dy} = f(\frac{x}{y})$ can be solve by making the substitution

(a) y = vx (b) v = yx (c) x = vy (d) x = v

9. Order and degree of differential equation are always

(a) positive integer (b) Negative integer (c) integer (d) None of these

10. The differential equation of y = Acosx – Bsinx is

(a) y'' - y = 0 (b) y'' +y = 0 (c) y' +y = 0 (d) y'' +x y = 0

11. The differential equation of y = A$e^{x} + Be^{-x}$+3x is

(a) y'' +x y = 0 (b) y'' + y = 3x (c) y'' - y = - 3x (d) y'' + y = 2x

12. Order and degree of differential equation $(\frac{d^2y}{dx^2})^{\frac{1}{2}} = (x - \frac{dy}{dx})^{\frac{1}{3}}$

(a) (3,2) (b) (1,3) (c) (2,1) (d) (2,3)

13. The solution of the differential equation $\frac{dy}{dx} = \frac{y+1}{x-1}$ is

(a) c = $\frac{y+1}{x-1}$ (b) c = $\frac{y+1}{x+1}$ (c) c = (y+1)(x-1) (d) c(x-1) =y

14. The general solution of differential equation $\frac{dy}{dx}$ =y^2x^3

(a) y = x^3+k (b) y = x^4+k (c) y = $\frac{-4}{x^4+k}$ (d) y = $x^3+e^{2x} + k$

15. The solution of the differential equation $\frac{dy}{dx} = e^{2x-y} + x^3e^{-y}$ is

(a) x(2x-y) + y(3-y) = c (b) $4e^{y} = 2e^{2x} + x^4 + c$

(c) $2e^{2x} + e^{y} = c$ (d) $2e^{2x} + e^{y} + x^4 = c$

16. The general solution of differential equation sinxcosydx – cosxsinydy = 0

(a) sinx = xsecy (b) xsecx = csecy (c) secx = cscey (d) secx = scey + c

17. The particular solution of y' = xe^x with initial condition y = 3 when x = 1

(a) x = ye^x-e^x+3 (b) y = xe^x-e^x+1 (c) y =xe^x-e^x+3 (d) y = e^x(x+1) + 3

18. A curve passes through the point (0,0) with differential equation y' = e^{3x-2y} ,then the equation of the curve is

(a) $3e^x$ = ye^{3x}+1 (b) $2e^{2y}$ = $3e^{3x}$+1 (c) $3e^{2y}$ = $2e^{3x}$ (d) $3e^{2y}$ = $2e^{3x}$ + 2

19. The general solution of differential equation $\frac{dy}{dx} = e^x$

(a) y = e^x+c (b) y =e^{-x}+c (c) y =-e^x+c (d) x = logy+c

20. The differential eqution of the family parabolas $y^2 = 4ax$ is

(a) $\frac{dy}{dx}$ = 4 $(\frac{dy}{dx})^2$ (b) y = 2x$\frac{dy}{dx}$ (c) $\frac{d^2y}{dx^2}$ = 4 (d) none of these

21. The differential eqution of the family of straight lines is

(a) $\frac{dy}{dx}$ = 4 $(\frac{dy}{dx})^2$ (b) y = 2x$\frac{dy}{dx}$ (c) $\frac{d^2y}{dx^2}$ =0 (d) None of these

22. Differential equation of the family of circles with center at origin & radius a is

(a) X-y$\frac{dy}{dx}$ = 0 (b) y- x$\frac{dy}{dx}$ = 0 (c) x+y$\frac{dy}{dx}$ = 0 (d) y+x$\frac{dy}{dx}$ = 0

23.Differential equation of the family of circles which passes through the origin & whose centers are on the x-axis is

(a) 2xy$\frac{dy}{dx}$+x^2+y^2 = 0 (b) 2xy $\frac{dy}{dx}$+x^2- y^2= 0 (c) x $\frac{dy}{dx}$+x^2- y^2= 0 (d) 2y $\frac{dy}{dx}$+x^2- y^2= 0

24.The solution of (1+ $x^2)dy = (1 + y^2)dx$ is

(a) $tan^{-1}y - tan^{-1}x$ = $tan^{-1}c$

(b) $tan^{-1}y + tan^{-1}x$ = $tan^{-1}c$

(c) $tan^{-1}x - tan^{-1}y$ = $tan^{-1}c$

(d) $tan^{-1}(\frac{y}{x})$ = $tan^{-1}c$

25.Solution of (xy^2+x)dx+(x$y^2 + y)dy = 0$is

(a) $(x^2 + 1) = c(y^2 + 1)$

(b) $(x^2 - 1) = c(y^2 - 1)$

(c) $(x^2 + 1)(y^2 + 1)$ = c

(d) $(x^2 - 1)(y^2 - 1)$ = c