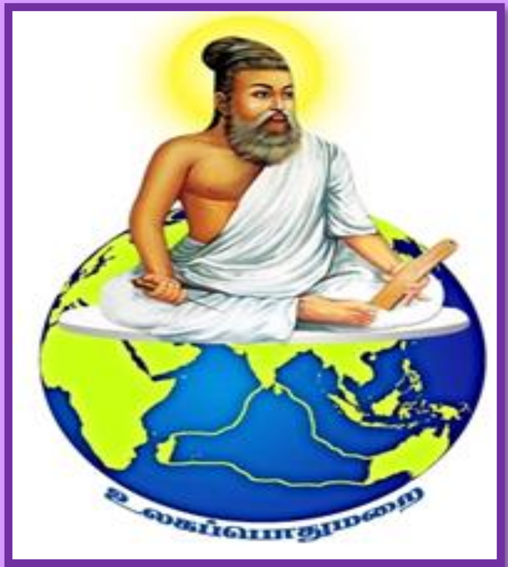
HIGHER SECONDARY
SECOND YEAR

PHYSICS



UNIT - 8

DUAL NATURE OF RADIATON AND MATTER

PROBLEMS AND SOLUTIONS

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EXAMPLE PROBLEMS

1. For the photoelectric emission from cesium, show that wave theory predicts that (i) maximum kinetic energy of the photoelectrons (K_{max}) depends on the intensity I of the incident light (ii) K_{max} does not depend on the frequency of the incident light and (iii) the time interval between the incidence of light and the ejection of photoelectrons is very long. (Given : The work function for cesium is 2.14 eV and the power absorbed per unit area is $1.60 \ X\ 10^{-6}\ W\ m^{-2}$ which produces a measurable photocurrent in cesium.)

Solution :

For the sake of simplicity, the following standard assumptions can be made when light is incident on the given material.

(a) Light is absorbed in the top atomic layer of the metal

(b) For a given element, each atom absorbs an equal amount of energy and this energy is proportional to its cross-sectional area A

(c) Each atom gives this energy to one of the electrons.

(i) According to wave theory, the energy in a light wave is spread out uniformly and continuously over the wavefront.

- The energy absorbed by each electron in time t is given by **E = I A t**
- With this energy absorbed, the most energetic electron is released with K_{max} by overcoming the surface energy barrier or work function ϕ_0 and this is expressed as ; $K_{max} = I\,A\,t - \phi_0$
- Thus, wave theory predicts that for a unit time, at low light intensities when $I\,A < \phi_0$, no electrons are emitted. At higher intensities, when $I\,A \geq \phi_0$, electrons are emitted. This implies that higher the light intensity, greater will be K_{max}.
- K_{max} is dependent only on the intensity under given conditions – that is, by suitably increasing the intensity, one can produce photoelectric effect even if the frequency is less than the threshold frequency. So the concept of threshold frequency does not even exist in wave theory.

(ii) According to wave theory, the intensity of a light wave is proportional to the square of the amplitude of the electric field (E_0^2). The amplitude of this electric field increases with increasing intensity and imparts an increasing acceleration and kinetic energy to an electron. This means that K_{max} should not depend at all on the frequency of the classical light wave which again contradicts the experimental results.

(iii) If an electron accumulates light energy just enough to overcome the work function, then it is ejected out of the atom with zero kinetic energy. Therefore,

$$0 = I\,A\,t - \phi_0 \quad (or) \quad I\,A\,t = \phi_0$$

$$\therefore \ t = \frac{\phi_0}{I\,A} = \frac{\phi_0}{I\,\pi\,r^2} = \frac{2.14\ X\ 1.6\ X\ 10^{-19}}{1.60\ X\ 10^{-6}\ X\ 3.14\ X\ (1\ X\ 10^{-10})^2} = \mathbf{6.8\ X\ 10^6\ s \approx 79\ days}$$

- Thus, wave theory predicts that there is a large time gap between the incidence of light and the ejection of photoelectrons but the experiments show that photo emission is an instantaneous process

2. A radiation of wavelength 300 nm is incident on a silver surface. Will photoelectrons be observed? [work function of silver = 4.7 eV]

Solution : : $\lambda = 300\ nm = 300\ X\ 10^{-9} m$

- Energy of the incident photon is

$$E = h\,\nu = \frac{h\,c}{\lambda}\ (joule)$$

$$E = \frac{h\,c}{\lambda\,e}\ (eV)$$

$$E = \frac{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^8}{300\ X\ 10^{-9}\ X\ 1.6\ X\ 10^{-19}}$$

$$E = \frac{6.626\ X\ 10^{-26}}{100\ X\ 10^{-28}\ X\ 1.6} = \frac{6.626}{1.6} = \frac{66.26}{16}$$

$$\mathbf{E = 4.14\ eV}$$

- The work function of silver = 4.7 eV. Since the energy of the incident photon is less than the work function of silver, photoelectrons are not observed in this case.

3. When light of wavelength 2200Å falls on Cu, photo electrons are emitted from it. Find (i) the threshold wavelength and (ii) the stopping potential. Given: the work function for Cu is ϕ_0 = 4.65 eV.

Solution : $\lambda = 2200\ A^\circ = 2200\ X\ 10^{-10} m$; $\phi_0 = 4.65\ eV = 4.65\ X\ 1.6\ X\ 10^{-19} J$

(i) Work function ; $\phi_0 = h\,\nu_0 = \frac{h\,c}{\lambda_0}$

Hence threshold wavelength,

$$\lambda_0 = \frac{h\,c}{\phi_0} = \frac{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^8}{4.65\ X\ 1.6\ X\ 10^{-19}} = \frac{19.878\ X\ 10^{-26}}{7.44\ X\ 10^{-19}}$$

$$\lambda_0 = \frac{19.878\ X\ 10^{-7}}{7.44}$$

$$\mathbf{\lambda_0 = 2.672\ X\ 10^{-7}\ m = 2672\ X\ 10^{-10}\ m = 2672\ A^\circ}$$

No	Log
19.878	1.2984
7.44	0.8716
(-)	0.4268
ALog	$2.672\ X\ 10^0$

(ii) By Einstein's photo electric equation; $K_{max} = h\nu - \phi_0$ (or) $e\,V_0 = h\nu - \phi_0$

- Energy of incident photon; $E = h\,\nu = \frac{h\,c}{\lambda}$

$$E = \frac{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^8}{2200\ X\ 10^{-10}} = \frac{19.878\ X\ 10^{-16}}{2200}$$

$$E = 9.036\ X\ 10^{-3}\ X\ 10^{-16} = 9.036\ X\ 10^{-19}\ J$$

No	Log
19.878	1.2984
2200	3.3424
(-)	3.9560
ALog	$9.036\ X\ 10^{-3}$

- and working function,

$$\phi_0 = 4.65\ eV = 4.65\ X\ 1.6\ X\ 10^{-19}\ J = 7.44\ X 10^{-19}\ J$$

- Hence, $e\,V_0 = h\nu - \phi_0$

$$e\,V_0 = 9.036\ X\ 10^{-19} - 7.44\ X 10^{-19} = 1.596\ X\ 10^{-19} J$$

$$(or) \quad e\,V_0 = 1.6\ X\ 10^{-19} J$$

- Then stopping potential,

$$V_0 = \frac{1.6\ X\ 10^{-19}}{e} = \frac{1.6\ X\ 10^{-19}}{1.6\ X\ 10^{-19}}$$

$$\mathbf{V_0 = 1\ V}$$

4. The work function of potassium is 2.30 eV. UV light of wavelength 3000 Å and intensity 2 Wm⁻² is incident on the potassium surface. (i) Determine the maximum kinetic energy of the photo electrons (ii) If 40% of incident photons produce photo electrons, how many electrons are emitted per second if the area of the potassium surface is 2 cm² ?

Solution : $\lambda = 3000\ A° = 3000\ X\ 10^{-10} m\ ;\ \phi_O = 2.30\ eV\ ;\ A = 2\ cm^2 = 2\ X\ 10^{-4}\ m^2$

(i) Energy of incident photon; $E = h\,\nu = \frac{h\,c}{\lambda}$

$$E = \frac{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^{8}}{3000\ X\ 10^{-10}} = \frac{6.626\ X\ 10^{-36}}{1000\ X\ 10^{-10}} = \mathbf{6.626\ X\ 10^{-19}\ J}$$

$$E = \frac{6.626\ X\ 10^{-19}}{1.6\ X\ 10^{-19}}\ eV = \frac{6.626}{1.6}\ eV = \frac{66.26}{16}\ eV = \mathbf{4.14\ eV}$$

By Einstien's photo electric equation, the maximum kinetic energy is,

$$K_{max} = h\nu - \phi_O$$

$$K_{max} = 4.14 - 2.30 = \mathbf{1.84\ e\ V} \qquad [\because 1\ eV = 1.6\ X\ 10^{-19}\ J]$$

$$(or) \qquad K_{max} = \mathbf{2.946\ X\ 10^{-19}}$$

(ii) Number of photons reaches the surface per second; $n_P = \frac{P}{E}\ X\ A$

$$n_P = \frac{2}{6.626\ X\ 10^{-19}}\ X\ 2\ X\ 10^{-4} = \frac{4}{6.626}\ X\ 10^{15}$$

$$n_P = 6.037\ X\ 10^{-1}\ X\ 10^{15} = \mathbf{6.037\ X\ 10^{14}\ photons/sec}$$

No	Log
4	0.6021
6.626	0.8213
(-)	$\bar{1}$.7808
ALog	6.037 X 10⁻¹

Hence rate of emission of photons,

$$n = 40\%\ n_P = 0.40\ X\ n_P = 0.4\ X\ 6.037\ X\ 10^{14}$$

$$\mathbf{n = 2.415\ X\ 10^{14}\ photons/sec}$$

5. Light of wavelength 390 nm is directed at a metal electrode. To find the energy of electrons ejected, an opposing potential difference is established between it and another electrode. The current of photoelectrons from one to the other is stopped completely when the potential difference is 1.10 V. Determine (i) the work function of the metal and (ii) the maximum wavelength of light that can eject electrons from this metal.

Solution : $\lambda = 390\ nm = 390\ X\ 10^{-9}\ m\ ;\ V_O = 1.10\ V$

No	Log
19.878	1.2984
390	2.5911
(-)	$\bar{2}$.7073
ALog	5.097 X 10⁻²

(i) By Einstien's photo electric equation,,

$$K_{max} = h\nu - \phi_O \quad (or) \quad e\,V_O = h\nu - \phi_O$$

♣ Hence working function,

$$\phi_O = h\nu - eV_O = \frac{hc}{\lambda} - eV_O$$

$$\phi_O = \left[\frac{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^{8}}{390\ X\ 10^{-9}}\right] - [1.6\ X\ 10^{-19}\ X\ 1.10]$$

$$\phi_O = \left[\frac{19.878\ X\ 10^{-17}}{390}\right] - [1.76\ X\ 10^{-19}]$$

$$\phi_O = [5.097\ X\ 10^{-19}] - [1.76\ X\ 10^{-19}] = [5.097 - 1.76]\ X\ 10^{-19}$$

$$\boldsymbol{\phi_O = 3.337\ X\ 10^{-19}\ J}$$

$$(or) \qquad \phi_O = \frac{3.337\ X\ 10^{-19}}{1.6\ X\ 10^{-19}}\ eV = \frac{33.37}{16}$$

$$\boldsymbol{\phi_O = 2.085\ eV}$$

(ii) Threshold wavelength ;

No	Log
19.878	1.2984
3.337	0.5234
(-)	0.7750
ALog	5.957 X 10⁰

$$\lambda_O = \frac{h\,c}{\phi_O} = \frac{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^{8}}{3.337\ X\ 10^{-19}}$$

$$\lambda_O = \frac{19.878\ X\ 10^{-26}}{3.337\ X\ 10^{-19}} = \frac{19.878\ X\ 10^{-7}}{3.337}$$

$$\boldsymbol{\lambda_O = 5.957\ X\ 10^{-7}\ m = 5957\ X\ 10^{-10}\ m = 5957\ A°}$$

6. Calculate the momentum and the de Broglie wavelength in the following cases:
(i) an electron with kinetic energy 2 eV.
(ii) a bullet of 50 g fired from rifle with a speed of 200 m/s
(iii) a 4000 kg car moving along the highways at 50 m/s
Hence show that the wave nature of matter is important at the atomic level but is not really relevant at macroscopic level.

Solution :

<u>(i)</u> $K = 2\ eV = 2\ X\ 1.6\ X\ 10^{-19}\ J\ ;\ m = 9.1`\ X\ 10^{-31} kg$

No	Log
√58.24	1.7652 X (1/2)
	0.8826
ALog	7.631 X 10⁰

Momentum of electron,

$$p = \sqrt{2\,m\,K}$$

$$p = \sqrt{2\ X\ 9.1`\ X\ 10^{-31}\ X\ 2\ X\ 1.6\ X\ 10^{-19}}$$

$$p = \sqrt{58.24\ X\ 10^{-50}} = \mathbf{7.631\ X\ 10^{-25}\ kg\ m\ s^{-1}}$$

Hence de Broglie wavelength of electron,

No	Log
6.626	0.8213
7.631	0.8826
(-)	$\bar{1}$.9387
ALog	8.684 X 10⁻¹

$$\lambda = \frac{h}{p} = \frac{6.626\ X\ 10^{-34}}{7.631\ X\ 10^{-25}}$$

$$\lambda = \frac{6.626\ X\ 10^{-9}}{7.631}$$

$$\lambda = 8.684\ X\ 10^{-1}\ X\ 10^{-9} = 8.684\ X\ 10^{-10}\ m$$

$$\boldsymbol{\lambda = 8.684\ A°}$$

<u>(ii)</u> $m = 50\ g = 50\ X\ 10^{-3} kg\ ;\ v = 200\ ms^{-1}$, then momentum of bullet,

$$p = m\,v = 50\ X\ 10^{-3}\ X\ 200 = 10000\ X\ 10^{-3}$$

$$\mathbf{p = 10\ \ kg\ m\ s^{-1}}$$

Hence de Broglie wavelength of bullet,

$$\lambda = \frac{h}{p} = \frac{6.626\ X\ 10^{-34}}{10} = \mathbf{6.626\ X\ 10^{-35}\ \ kg\ m\ s^{-1}}$$

<u>(iii)</u> $m = 4000\ kg\ ;\ v = 50\ ms^{-1}$ then momentum of car,

$$p = m\,v = 4000\ X\ 50 = 200000$$

$$\mathbf{p = 2\ X\ 10^{5}\ \ kg\ m\ s^{-1}}$$

Hence de Broglie wavelength of car,

$$\lambda = \frac{h}{p} = \frac{6.626\ X\ 10^{-34}}{2\ X\ 10^{5}} = \mathbf{3.313\ X\ 10^{-39}\ \ kg\ m\ s^{-1}}$$

♣ From these calculations, we notice that electron has significant value of de Broglie wavelength (≈10^{-9}m which can be measured from diffraction studies) but moving bullet and car have negligibly small de Broglie wavelengths associated with them (≈10^{-33}m and 10^{-39}m respectively, which are not measurable by any experiment).

♣ This implies that the wave nature of matter is important at the atomic level but it is not really relevant at the macroscopic level.

7. Find the de Broglie wavelength associated with an alpha particle which is accelerated through a potential difference of 400 V. Given that the mass of the proton is 1.67 × 10^{-27} kg.

Solution : $V = 400\ V$; $m_P = 1.67\ X\ 10^{-27} kg$

- An alpha particle contains 2 protons and 2 neutrons. It is represented by ${}^4_2He$
- Hence ; $q = 2e = 2\ X\ 1.6\ X\ 10^{-19} = 3.2\ X\ 10^{-19}\ C$

$$M = 4\ m_P = 4\ X\ 1.67\ X\ 10^{-27} = 6.68\ X\ 10^{-27}\ kg$$

- The de Broglie wavelength associated with it is,

$$\lambda = \frac{h}{\sqrt{2\ M\ q\ V}}$$

$$\lambda = \frac{6.626\ X\ 10^{-34}}{\sqrt{2\ X\ 6.68\ X\ 10^{-27} X\ 3.2\ X\ 10^{-19}\ X\ 400}}$$

$$\lambda = \frac{6.626\ X\ 10^{-34}}{\sqrt{17100.8\ X\ 10^{-46}}} = \frac{6.626\ X\ 10^{-11}}{\sqrt{17100.8}}$$

$$\lambda = 5.068\ X\ 10^{-2}\ X 10^{-11}$$

$$\boldsymbol{\lambda = 5.068\ X\ 10^{-13}\ m = 0.005068\ X\ 10^{-10}\ m = 0.005068\ A^{\circ}}$$

No	Log
6.626	0.8213
17100.8	4.2330 X(1/2)
	2.1165
Nr	0.8213
Dr	2.1165
(-)	$\bar{2}.7048$
ALog	5.068 X 10^{-2}

8. A proton and an electron have same de Broglie wavelength. Which of them moves faster and which possesses more kinetic energy?

Solution :

- de Broglie wavelength of proton ; $\lambda_P = \frac{h}{\sqrt{2\ m_P\ K_P}}$
- de Broglie wavelength of electron ; $\lambda_e = \frac{h}{\sqrt{2\ m_e\ K_e}}$
- Since proton and electron have same de Broglie wavelength ($\lambda_P = \lambda_e$), we get,

$$\frac{h}{\sqrt{2\ m_P\ K_P}} = \frac{h}{\sqrt{2\ m_e\ K_e}}$$

$$(or) \qquad \frac{K_P}{K_e} = \frac{m_e}{m_P} \qquad --------- \quad (1)$$

Since $\boldsymbol{m_e} < \boldsymbol{m_P}$; $\boldsymbol{K_P} < \boldsymbol{K_e}$ $(i.e.)$ the electron has more kinetic energy than the proton.

- Also kinetic energy of proton ; $K_P = \frac{1}{2}\ m_P v_P^2$
- And kinetic energy of eletron ; $K_e = \frac{1}{2}\ m_e v_e^2$
- Then ratio of the kinetic energies ; $\frac{K_P}{K_e} = \frac{\frac{1}{2} m_P v_P^2}{\frac{1}{2} m_e v_e^2}$

$$\frac{v_P^2}{v_e^2} = \frac{K_P}{K_e}\frac{m_e}{m_P} \quad (or) \quad \frac{v_P}{v_e} = \sqrt{\frac{K_P}{K_e}\frac{m_e}{m_P}}$$

- Put equation (1), $\frac{v_P}{v_e} = \sqrt{\frac{m_e}{m_P}\frac{m_e}{m_P}} = \sqrt{\frac{m_e^2}{m_P^2}} = \boldsymbol{\frac{m_e}{m_P}}$

Since $\boldsymbol{m_e} < \boldsymbol{m_P}$; $\boldsymbol{v_P} < \boldsymbol{v_e}$ $(i.e.)$ the electron moves faster than the proton.

9. Calculate the cut-off wavelength and cut-off frequency of x-rays from an x –ray tube of accelerating potential 20,000 V.

Solution : $V = 20000\ V$

- The cut-off wavelength of the x-rays in the continuous spectrum is given by,

$$\lambda_O = \frac{12400}{V}\ A^{\circ}$$

$$\lambda_O = \frac{12400}{20000}\ A^{\circ} = \frac{12400}{2\ X\ 10^4}\ A^{\circ} = 6200\ X\ 10^{-4}\ A^{\circ}$$

$$\boldsymbol{\lambda_O = 0.62\ A^{\circ}}$$

- The corresponding frequency is

$$\nu_O = \frac{c}{\lambda_O}$$

$$\nu_O = \frac{3\ X\ 10^8}{0.62\ X\ 10^{-10}} = \frac{3\ X\ 10^{18}}{0.62}$$

$$\boldsymbol{\nu_O = 4.838\ X\ 10^{18}\ Hz}$$

No	Log
3	0.4771
0.62	$\bar{1}.7924$
(-)	0.6847
ALog	4.838 X 10^0

EXERCISE PROBLEMS

1. How many photons per second emanate from a 50 mW laser of 640 nm?

Solution : $P = 50\ mW = 50\ X\ 10^{-3}\ W$; $\lambda = 640\ nm = 640\ X\ 10^{-9}\ m$

♣ Number of photons per second,

$$n_P = \frac{P}{E} = \frac{P}{h\nu} = \frac{P}{\left(^{hc}/_{\lambda}\right)} = \frac{P\lambda}{hc}$$

$$n_P = \frac{50\ X\ 10^{-3}\ X\ 640\ X\ 10^{-9}}{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^{8}} = \frac{32000\ X\ 10^{-12}}{19.878\ X\ 10^{-26}}$$

$$n_P = \frac{32000\ X\ 10^{14}}{19.878} = 1.610\ X\ 10^{3}\ X\ 10^{14} = \mathbf{1.61\ X\ 10^{17}}$$

No	Log
32000	4. 5051
19.878	1. 2984
(-)	3. 2067
ALog	1. 610 X 10^3

2. Calculate the maximum kinetic energy and maximum velocity of the photoelectrons emitted when the stopping potential is 81 V for the photoelectric emission experiment.

Solution : $V_O = 81\ V$

♣ The maximum kinetic energy of photo electrons is equal to stopping potential energy. (i.e.)

$$K_{max} = e\,V_O$$
$$K_{max} = 1.6\ X\ 10^{-19}\ X\ 81 = 129.6\ X\ 10^{-19}$$
$$\mathbf{K_{max} = 1.296\ X\ 10^{-17}\ J}$$

♣ But kinetic energy is given by

$$K_{max} = \frac{1}{2}\,m v_{max}^2$$

$$1.296\ X\ 10^{-17} = \frac{1}{2}\ X\ 9.1\ X\ 10^{-31}\ X\ v_{max}^2$$

$$v_{max}^2 = \frac{2\ X\ 1.296\ X\ 10^{-17}}{9.1\ X\ 10^{-31}} = \frac{2.592\ X\ 10^{14}}{9.1}$$

$$\therefore\quad v_{max} = \sqrt{\frac{2.592\ X\ 10^{14}}{9.1}} = \sqrt{\frac{259.2\ X\ 10^{12}}{9.1}}$$

$$\mathbf{v_{max} = 5.337\ X 10^{6}\ m\ s^{-1}}$$

No	Log
259.2	2. 4136
9.1	0. 9590
(-)	1. 4546 X (1/2)
	0. 7273
ALog	5. 337 X 10^0

3. Calculate the energies of the photons associated with the following radiation: (i) violet light of 413 nm (ii) X-rays of 0.1 nm (iii) radio waves of 10 m.

Solution :

(i) If $\lambda = 413\ nm = 413\ X\ 10^{-9}\ m$, then energy of violet light photon,

$$E = h\nu = \frac{hc}{\lambda}$$

$$E = \frac{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^{8}}{413\ X\ 10^{-9}} = \frac{19.878\ X\ 10^{-36}}{413\ X\ 10^{-9}}$$

$$E = \frac{19.878\ X\ 10^{-17}}{413}\ J$$

(or) $$E = \frac{19.878\ X\ 10^{-17}}{413\ X\ 1.6\ X\ 10^{-19}}\ eV = \frac{19.878\ X\ 10^{2}}{660.8}\ eV$$

$$E = 3.008\ X\ 10^{-2}\ X\ 10^{2} = \mathbf{3.008\ eV} \approx \mathbf{3\ eV}$$

(ii) If $\lambda = 0.1\ nm = 0.1\ X\ 10^{-9}\ m$ then, energy of X-ray photon,

$$E = h\nu = \frac{hc}{\lambda}$$

$$E = \frac{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^{8}}{0.1\ X\ 10^{-9}} = \frac{19.878\ X\ 10^{-36}}{1\ X\ 10^{-10}}$$

$$E = 19.878\ X\ 10^{-16}\ J$$

(or) $$E = \frac{19.878\ X\ 10^{-17}}{1.6\ X\ 10^{-19}}\ eV = \frac{19.878\ X\ 10^{2}}{1.6}\ eV = \frac{198.78\ X\ 10^{2}}{16}\ eV$$

$$E = 12.42\ X\ 10^{2}\ \mathbf{eV} = \mathbf{1242\ eV}$$

(iii) If $\lambda = 10\ m$ then, energy of radio waves,

$$E = h\nu = \frac{hc}{\lambda}$$

$$E = \frac{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^{8}}{10} = \frac{19.878\ X\ 10^{-36}}{10}$$

$$E = 19.878\ X\ 10^{-27}\ J$$

(or) $$E = \frac{19.878\ X\ 10^{-27}}{1.6\ X\ 10^{-19}}\ eV = \frac{19.878\ X\ 10^{-8}}{1.6}\ eV = \frac{198.78\ X\ 10^{-8}}{16}\ eV$$

$$E = 12.42\ X\ 10^{-8} = \mathbf{1.242\ X\ 10^{-7}\ eV}$$

4. A 150 W lamp emits light of mean wavelength of 5500 Å . If the efficiency is 12%, find out the number of photons emitted by the lamp in one second.

Solution : $P = 150\ W$; $\lambda = 5500\ A^{\circ} = 5500\ X\ 10^{-10} m$; $\eta = 12\% = \frac{12}{100}$

♣ Number of photons emitted per second,

$$n_P = \frac{P}{E} = \frac{P}{h\nu} = \frac{P}{\left(^{hc}/_{\lambda}\right)} = \frac{P\lambda}{hc}$$

$$n_P = \frac{150\ X\ 5500\ X\ 10^{-10}}{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^{8}}$$

$$n_P = \frac{825000\ X\ 10^{-10}}{19.878\ X\ 10^{-26}} = \frac{825\ X\ 10^{19}}{19.878} = 4.150\ X\ 10^{1}\ X\ 10^{19}$$

$$\mathbf{n_P = 4.150\ X\ 10^{20}\ photons/sec}$$

No	Log
825	2. 9165
19.878	1. 2984
(-)	1. 6181
ALog	4. 150 X 10^1

♣ The number of photons emitted by the lamp in one second,

$$n = \eta\, n_P = \frac{12}{100}\ X\ 4.150\ X\ 10^{20} = 12\ X\ 4.150\ X\ 10^{18} = 49.8\ X\ 10^{18}$$

$$\mathbf{n = 4.98\ X\ 10^{19}\ photons/sec}$$

5. How many photons of frequency 10^{14} Hz will make up 19.86 J of energy?

Solution : $\nu = 10^{14}\ Hz$; $P = \frac{U}{t} = 19.86\ J$

♣ Number of photons emitted per second,

$$n_P = \frac{P}{E} = \frac{P}{h\nu}$$

$$n_P = \frac{19.86}{6.626\ X 10^{-34}\ X\ 10^{14}}$$

$$n_P = \frac{19.86\ X 10^{20}}{6.626}$$

$$\mathbf{n_P = 2.997\ X\ 10^{20}} \approx \mathbf{3\ X\ 10^{20}}$$

No	Log
19.86	1. 2980
6.626	0. 8213
(-)	0. 4767
ALog	2. 997 X 10^0

6. What should be the velocity of the electron so that its momentum equals that of 4000 Å wavelength photon.

Solution : $p_e = p_p$; $\lambda_p = 4000\ A° = 4000\ X\ 10^{-10}\ m$

No	Log
6.626	0.8213
36.4	1.5611
(-)	1.2602
ALog	1.821 X 10^{-1}

♣ de Broglie wavelength of photon,

$$\lambda_p = \frac{h}{p_p} = \frac{h}{p_e} = \frac{h}{m\,v_e}$$

$$\therefore \quad v_e = \frac{h}{m\,\lambda_p} = \frac{6.626\ X 10^{-34}}{9.1\ X\ 10^{-31}\ X\ 4000\ X\ 10^{-10}}$$

$$v_e = \frac{6.626\ X 10^{7}}{9.1\ X\ 4000} = \frac{6.626\ X 10^{4}}{36.4} = 1.821\ X\ 10^{-1}\ X\ 10^{4}$$

$$\boldsymbol{v_e = 1821\ m\ s^{-1}}$$

7. When a light of frequency 9 X 10^{14} Hz is incident on a metal surface, photoelectrons are emitted with a maximum speed of 8 X10^5 ms^{-1}. Determine the threshold frequency of the surface.

Solution : $: \nu = 9\ X\ 10^{14}\ Hz$; $v_{max} = 8\ X\ 10^5\ m\ s^{-1}$

No	Log
30.514	1.4844
6.626	0.8213
(-)	0.6631
ALog	4.603 X 10^0

♣ By Einstein's photo electric equation,

$$h\,\nu = h\,\nu_O + \frac{1}{2}\,m\,v_{max}^2$$

$$(or) \quad h\,\nu_O = h\,\nu - \frac{1}{2}\,m\,v_{max}^2$$

$$h\,\nu_O = [6.626\ X 10^{-34} X 9\ X\ 10^{14}] - \left[\frac{1}{2}\ X\ 9.1\ X\ 10^{-31} X\ 64\ X\ 10^{10}\right]$$

$$h\,\nu_O = [59.634\ X 10^{-20}] - [291.2\ X\ 10^{-21}]$$

$$(or) \quad h\,\nu_O = [59.634\ X 10^{-20}] - [29.12\ X\ 10^{-20}]$$

$$h\,\nu_O = [59.634 - 29.12]\ X 10^{-20} = 30.514\ X 10^{-20}$$

$$\therefore \quad \nu_O = \frac{30.514\ X 10^{-20}}{h} = \frac{30.514\ X 10^{-20}}{6.626\ X 10^{-34}} = \frac{30.514\ X 10^{14}}{6.626}$$

$$\boldsymbol{\nu_O = 4.603\ X 10^{14}\ Hz}$$

8. When a 6000Å light falls on the cathode of a photo cell, photoemission takes place. If a potential of 0.8 V is required to stop emission of electron, then determine the (i) frequency of the light (ii) energy of the incident photon (iii) work function of the cathode material (iv) threshold frequency and (v) net energy of the electron after it leaves the surface.

Solution : $\lambda = 6000\ A° = 6000\ X\ 10^{-10}\ m$; $V_O = 0.8\ V$

(i) Frequency of light ,

$$\nu = \frac{c}{\lambda} = \frac{3\ X\ 10^8}{6000\ X\ 10^{-10}} = \frac{1\ X\ 10^{15}}{2} = 0.5\ X\ 10^{15}$$

$$\boldsymbol{\nu = 5\ X\ 10^{14}\ Hz}$$

(ii) Energy of incident photon,

$$E = h\,\nu = 6.626\ X 10^{-34}\ X\ 5\ X\ 10^{14} = 33.13\ X\ 10^{-20}\ J$$

$$(or) \quad E = \frac{33.13\ X\ 10^{-20}}{1.6\ X\ 10^{-19}}\ eV = \frac{331.3\ X\ 10^{-1}}{16}\ eV = 20.71\ X\ 10^{-1} eV$$

$$\boldsymbol{E = 2.071\ eV}$$

(iii) By Einstein's photo electric equation,

$$h\,\nu = \phi_O + \frac{1}{2}\,m\,v_{max}^2$$

$$(or) \quad \phi_O = h\,\nu - \frac{1}{2}\,m\,v_{max}^2$$

$$(or) \quad \phi_O = h\,\nu - e\,V_O$$

$$\phi_O = (33.13\ X\ 10^{-20}) - (1.6\ X\ 10^{-19} X\ 0.8)$$

$$\phi_O = (33.13\ X\ 10^{-20}) - (1.28\ X\ 10^{-19})$$

$$\phi_O = (3.313\ X\ 10^{-19}) - (1.28\ X\ 10^{-19})$$

$$\phi_O = (3.313 - 1.28)\ X\ 10^{-19}$$

$$\boldsymbol{\phi_O = 2.033\ X\ 10^{-19}\ J}$$

$$(or) \quad \phi_O = \frac{2.033\ X\ 10^{-19}}{1.6\ X\ 10^{-19}}\ eV = \frac{20.33}{16}\ eV$$

$$(or) \quad \boldsymbol{\phi_O = 1.270\ eV}$$

No	Log
2.033	0.3081
6.626	0.8213
(-)	$\bar{1}$.4868
ALog	3.068 X 10^{-1}

(iv) Work function,

$$\phi_O = h\,\nu_O$$

$$(or) \quad \nu_O = \frac{\phi_O}{h} = \frac{2.033\ X\ 10^{-19}}{6.626\ X 10^{-34}} = \frac{2.033\ X\ 10^{15}}{6.626}$$

$$\boldsymbol{\nu_O} = 3.068\ X\ 10^{-1} X\ 10^{15} = \boldsymbol{3.068\ X\ 10^{14}\ H}$$

(v) Net energy of the electron after it leaves the surface is nothing but its kinetic energy which is given by,

$$K_{max} = h\nu - \phi_O$$

$$K_{max} = 2.071 - 1.270$$

$$\boldsymbol{K_{max} = 0.801\ eV}$$

9. A 3310 Å photon liberates an electron from a material with energy 3 X 10^{-19} J while another 5000 Å photon ejects an electron with energy 0.972 X 10^{-19} J from the same material. Determine the value of Planck's constant and the threshold wavelength of the material.

Solution : $\lambda_1 = 3310\ A° = 3310\ X\ 10^{-10}\ m$; $K_1 = 3\ X\ 10^{-19}$ J

$\lambda_2 = 5000\ A° = 5000\ X\ 10^{-10}\ m$; $K_2 = 0.972\ X\ 10^{-19}$ J

♣ By Einstein's photo electric equation,

$$h\,\nu = \phi_O + K_{max}$$

$$(or) \quad \frac{h\,c}{\lambda} = \phi_O + K_{max}$$

♣ For given material, work function is constant ,

$$\frac{h\,c}{\lambda_1} = \phi_O + K_1 \qquad ------ (1)$$

$$\frac{h\,c}{\lambda_2} = \phi_O + K_2 \qquad ------ (2)$$

$$(1) - (2) \Rightarrow \quad \frac{h\,c}{\lambda_1} - \frac{h\,c}{\lambda_2} = \phi_O + K_1 - \phi_O - K_2$$

$$h\,c\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = K_1 - K_2$$

No	Log
2.028	0.3071
16550	4.2188
(+)	4.5259
5070	3.7050
(-)	0.8209
ALog	6.621 X 10^0

$$h\,c\left[\frac{\lambda_2-\lambda_1}{\lambda_1\lambda_2}\right] = K_1 - K_2$$

$$h\,(3X10^8)\left[\frac{5000-3310}{3310\ X\ 5000}\right]\frac{1}{10^{-10}} = (3-0.972)\ X\ 10^{-19}$$

$$h\,(3X10^8)\left[\frac{1690}{16550\ X\ 10^{-7}}\right] = 2.028\ X10^{-19}$$

$$h = \frac{2.028\ X10^{-19}\ X\ 16550\ X\ 10^{-7}}{1690\ X\ 3X10^8}$$

$$h = \frac{2.028\ X16550\ X\ 10^{-34}}{5070}$$

$$\boldsymbol{h = 6.621\ X\ 10^{-34}\ J\ s}$$

♣ From equation (2), Work function is,

$$\phi_O = \frac{h\,c}{\lambda_2} - K_2$$

$$\phi_O = \left[\frac{6.621\ X\ 10^{-34}\ X\ 3X10^8}{5000\ X\ 10^{-10}}\right] - [0.972\ X\ 10^{-19}]$$

$$\phi_O = \left[\frac{19.863\ X\ 10^{-19}}{5}\right] - [\,0.972\ X\ 10^{-19}]$$

$$\phi_O = [3.972\ X\ 10^{-19}] - [0.972\ X\ 10^{-19}]$$

$$\phi_O = [3.972 - 0.972\,]\ X\ 10^{-19}$$

$$\phi_O = 3\ X\ 10^{-19}$$

$$(or)\qquad \frac{h\,c}{\lambda_O} = 3\ X\ 10^{-19}$$

$$\therefore\qquad \lambda_O = \frac{h\,c}{3\ X\ 10^{-19}} = \frac{6.621\ X\ 10^{-34}\ X\ 3\ X\ 10^8}{3\ X\ 10^{-19}}$$

$$\boldsymbol{\lambda_O = 6.621\ X\ 10^{-7}m = 6621\ X\ 10^{-10}\ m = 6621\ A^\circ}$$

10. At the given point of time, the earth receives energy from sun at 4 *cal* cm^{-2} min^{-1} Determine the number of photons received on the surface of the Earth per cm^2 per minute. (Given : Mean wavelength of sun light = 5500 Å)

Solution : $P = 4\ cal\ cm^{-2}min^{-1} = 4\ X\ 4.2 = 16.8\ \ J\ cm^{-2}min^{-1}$

$\lambda = 5500\ A^\circ = 5500\ X\ 10^{-10}m$

♣ The number of photons received on the surface of the Earth per cm^2 per minute ,

$$n_P = \frac{P}{E} = \frac{P}{h\,\nu} = \frac{P}{\left(h\,c/_{\lambda}\right)} = \frac{P\,\lambda}{h\,c}$$

$$n_P = \frac{16.8\ \ X\ 5500\ X\ 10^{-10}}{6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^8}$$

$$n_P = \frac{924\ X\ 10^{18}}{19.878} = 4.648\ X\ 10^1X10^{18}$$

$$\boldsymbol{n_P = 4.648\ X\ 10^{19}}$$

No	Log
924 19.878	2. 9657 1. 2984
(-) ALog	1. 6673 4. 648 X 10^1

11. UV light of wavelength 1800Å is incident on a lithium surface whose threshold wavelength is 4965Å. Determine the maximum energy of the electron emitted.

Solution : $\lambda = 1800\ A^\circ = 1800\ X\ 10^{-10}m\ \ ;\ \ \lambda_O = 4965\ A^\circ = 4965\ X\ 10^{-10}m$

♣ By Einstein's photo electric equation,

$$h\,\nu = \phi_O + K_{max}$$

$$(or)\qquad K_{max} = h\,\nu - \phi_O$$

$$(or)\qquad K_{max} = h\,\nu - h\,\nu_O$$

$$(or)\qquad K_{max} = \frac{h\,c}{\lambda} - \frac{hc}{\lambda_O} = h\,c\left[\frac{1}{\lambda} - \frac{1}{\lambda_O}\right]$$

$$K_{max} = h\,c\left[\frac{\lambda_O - \lambda}{\lambda\,\lambda_O}\right]$$

$$K_{max} = 6.626\ X\ 10^{-34}\ X\ 3\ X\ 10^8\left[\frac{4965-1800}{4965\ X\ 1800}\right]X\,\frac{1}{10^{-10}}$$

$$K_{max} = 19.878\ X\ 10^{-19}\left[\frac{3165}{8937}\right]\ J$$

$$(or)\qquad K_{max} = \frac{19.878\ X\ 10^{-19}}{1.6\ X\ 10^{-19}}\left[\frac{3165}{8937}\right]\ eV$$

$$(or)\qquad K_{max} = \left[\frac{198.78}{16}\right]\left[\frac{3165}{8937}\right]\ eV$$

$$(or)\qquad K_{max} = \frac{12.42\ X\ 3165}{8937}\ \ eV$$

$$\boldsymbol{K_{max} = 4.398\ \ eV\ \approx 4.4\ eV}$$

No	Log
12.42 3165	1. 0941 3. 5004
(+) 8937	4. 5945 3. 9512
(-) ALog	0. 6433 4. 398 X 10^0

12. Calculate the de Broglie wavelength of a proton whose kinetic energy is equal to 81.9 × 10^{-15} J. (Given: mass of proton is 1836 times that of electron).

Solution : $K_P = 81.9\ \text{X}\ 10^{-15}\ \text{J}\ \ ;\ \ m_P = 1836\ m_e$

♣ de Broglie wavelength of proton,

$$\lambda_P = \frac{h}{\sqrt{2\ m_P\ K_P}} = \frac{h}{\sqrt{2\ (1836\ m_e)\ K_P}}$$

$$\lambda_P = \frac{6.621\ X\ 10^{-34}}{\sqrt{2\ X\ 1836\ X\ 9.1\ X\ 10^{-31}\ X\ 81.9\ X\ 10^{-15}}}$$

$$\lambda_P = \frac{6.621\ X\ 10^{-11}}{\sqrt{2\ X\ 1836\ X\ 9.1\ X\ 81.9}}$$

$$\lambda_P = 4.005\ X\ 10^{-3}X\ 10^{-11}$$

$$\boldsymbol{\lambda_P \approx 4\ X\ 10^{-14}\ m}$$

No	Log
6.626	0. 8213
2 1836 9,1 81.9	0. 3010 3. 2639 0. 9590 1. 9133
(+)	6. 4372 X (1/2) 3. 1186
Nr Dr	0. 8213 3. 1186
(-) ALog	3. 6027 4. 005 X 10^{-3}

13. A deuteron and an alpha particle are accelerated with the same potential. Which one of the two has (i) greater value of de Broglie wavelength associated with it and (ii) less kinetic energy? Explain.

Solution : $m_N = 1.67\ X\ 10^{-27}\ kg\ \ ;\ \ e = 1.6\ X\ 10^{-19}\ C$

♣ For deuteron ; $m_d = 2\ m_N\ ,\ \ q_d = e$

For alpha particle ; $m_\alpha = 4\ m_N\ ,\ \ q_\alpha = 2\ e$

(i) de Broglie wavelength of deuteron,

$$\lambda_d = \frac{h}{\sqrt{2\ m_d\ q_d\ V}} = \frac{h}{\sqrt{2\ (2\ m_N)\ e\ \ V}} = \frac{h}{\sqrt{4\ m_N\ e\ \ V}} = \frac{h}{2\ \sqrt{m_N\ e\ \ V}}$$

de Broglie wavelength of alpha particle,

$$\lambda_\alpha = \frac{h}{\sqrt{2\, m_\alpha\, q_\alpha\, V}} = \frac{h}{\sqrt{2\,(4\, m_N)\,(2\, e)\ V}} = \frac{h}{\sqrt{16\ m_N\, e\ V}} = \frac{h}{4\,\sqrt{m_N\, e\ V}}$$

$$\frac{\lambda_d}{\lambda_\alpha} = \frac{\left[\frac{h}{2\,\sqrt{m_N\, e\ V}}\right]}{\left[\frac{h}{4\,\sqrt{m_N\, e\ V}}\right]} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{1}{4}\right)} = \frac{4}{2} = 2$$

$$\boldsymbol{\lambda_d = 2\,\lambda_\alpha}$$

(ii) de Broglie wavelength of deuteron ,

$$\lambda_d = \frac{h}{\sqrt{2\, m_d\, K_d}} = \frac{h}{\sqrt{2\,(2\, m_N)\ K_d}} = \frac{h}{\sqrt{4\, m_N\, K_d}}$$

$$(or)\quad \lambda_d^2 = \frac{h^2}{4\, m_N\, K_d}$$

$$(or)\quad K_d = \frac{h^2}{4\, m_N\, \lambda_d^2}$$

de Broglie wavelength of alpha particle,

$$\lambda_\alpha = \frac{h}{\sqrt{2\, m_\alpha\, K_\alpha}} = \frac{h}{\sqrt{2\,(4\, m_N)\, K_\alpha}} = \frac{h}{\sqrt{8\, m_N\, K_\alpha}}$$

$$(or)\quad \lambda_\alpha^2 = \frac{h^2}{8\, m_N\, K_\alpha}$$

$$(or)\quad K_\alpha = \frac{h^2}{8\, m_N\, \lambda_\alpha^2}$$

$$\therefore\quad \frac{K_d}{K_\alpha} = \frac{\left[\frac{h^2}{4\, m_N\, \lambda_d^2}\right]}{\left[\frac{h^2}{8\, m_N\, \lambda_\alpha^2}\right]} = \frac{h^2}{4\, m_N\, \lambda_d^2}\ X\ \frac{8\, m_N\, \lambda_\alpha^2}{h^2} = 2\ \frac{\lambda_\alpha^2}{\lambda_d^2} = 2\left(\frac{\lambda_\alpha}{\lambda_d}\right)^2$$

$$\therefore\quad \frac{K_d}{K_\alpha} = 2\left(\frac{\lambda_\alpha}{2\,\lambda_\alpha}\right)^2 = 2\ X\ \frac{1}{4} = \frac{1}{2}$$

$$\boldsymbol{K_d = \frac{K_\alpha}{2}}$$

14. An electron is accelerated through a potential difference of 81V. What is the de Broglie wavelength associated with it? To which part of electromagnetic spectrum does this wavelength correspond?

Solution : $V = 81\ V$

♣ de Broglie wavelength of electron,

$$\lambda = \frac{12.27}{\sqrt{V}}\ A^\circ$$

$$\lambda = \frac{12.27}{\sqrt{81}}\ A^\circ = \frac{12.27}{9}\ A^\circ$$

$$\boldsymbol{\lambda = 1.36\ A^\circ}$$

♣ It lies in X -ray region of electromagnetic spectrum..

15. The ratio between the de Broglie wavelength associated with proton, accelerated through a potential of 512 V and that of alpha particle accelerated through a potential of X volts is found to be one. Find the value of X.

Solution : $V_P = 512\ V$; $\lambda_P : \lambda_\alpha = 1$; $V_\alpha = X$

♣ For proton ; $m_P = m_N$, $q_P = e$

For alpha particle ; $m_\alpha = 4\, m_N$, $q_\alpha = 2\, e$

♣ de Broglie wavelength of proton,

$$\lambda_P = \frac{h}{\sqrt{2\, m_P\ q_P\ V_P}} = \frac{h}{\sqrt{2\, m_N\, e\ V_P}}$$

de Broglie wavelength of alpha particle,

$$\lambda_\alpha = \frac{h}{\sqrt{2\, m_\alpha\, q_\alpha\, V_\alpha}} = \frac{h}{\sqrt{2\,(4\, m_N)\,(2\, e)\, X}} = \frac{h}{\sqrt{16\ m_N\, e\ X}}$$

$$\frac{\lambda_P}{\lambda_\alpha} = \frac{\left[\frac{h}{\sqrt{2\, m_N\, e\ V_P}}\right]}{\left[\frac{h}{\sqrt{16\, m_N\, e\ X}}\right]} = \frac{h}{\sqrt{2\, m_N\, e\ V_P}}\ X\ \frac{\sqrt{16\, m_N\, e\ X}}{h} = \sqrt{\frac{8\,X}{V_P}}$$

$$(or)\quad \left(\frac{\lambda_P}{\lambda_\alpha}\right)^2 = \frac{8\,X}{V_P}$$

$$(or)\quad X = \left(\frac{\lambda_P}{\lambda_\alpha}\right)^2 \frac{V_P}{8}$$

$$X = (1)^2\ \frac{512}{8}$$

$$\boldsymbol{X = 64\ V}$$