HIGHER SECONDARY
SECOND YEAR

PHYSICS

UNIT -7
WAVE OPTICS

PROBLEMS AND SOLUTIONS

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EXAMPLE PROBLEMS

1. The wavelength of light from sodium source in vacuum is 5893Å.What are its (a) wavelength, (b) speed and (c) frequency when this light travels in water which has a refractive index of 1.33.

Solution : $n_1 = 1$; $n_2 = 1.33$; $\lambda_1 = 5893\ A°$;

- Refractive index of water,

$$n = \frac{c}{v} = \frac{\lambda_1\, \nu}{\lambda_2\, \nu} = \frac{\lambda_1}{\lambda_2}$$

$$\lambda_2 = \frac{\lambda_1}{n} = \frac{5893\ X\ 10^{-10}}{1.33}$$

$$\boldsymbol{\lambda_2 = 4430\ X\ 10^{-10} = 4430\ A°}$$

No	Log
5893	3. 7703
1.33	0. 1239
(-)	3. 6464
Alog	4. 430 X 10^3

- Since, $n = \frac{c}{v}$, velocity of light in water,

$$v = \frac{c}{n} = \frac{3\ X\ 10^8}{1.33} = \frac{3\ X\ 10^8}{\left(\frac{4}{3}\right)} = \frac{9\ X\ 10^8}{4} = \mathbf{2.25\ X\ 10^8\ m\ s^{-1}}$$

- Frequency is same in both air and water medium, then

$$\nu = \frac{c}{\lambda_1} = \frac{3\ X\ 10^8}{5893\ X\ 10^{-10}} = \frac{3\ X\ 10^{18}}{5893}$$

$$\nu = 5.090\ X\ 10^{-4}\ X\ 10^{18} = \mathbf{5.090\ X\ 10^{14}\ Hz}$$

No	Log
3	0. 4771
5893	3. 7703
(-)	$\bar{4}$. 7068
ALog	5. 090 X 10^{-4}

2. Two light sources with amplitudes 5 units and 3 units respectively interfere with each other. Calculate the ratio of maximum and minimum intensities.

Solution : $a_1 = 5$; $a_2 = 3$

- Resultant amplitude,

$$A = \sqrt{a_1^2 + a_2^2 + 2\, a_1 a_2 \cos\phi}$$

- When $\phi = 0$ (or) $\cos\phi = 1$, then resultant amplitude will be maximum.

$$A_{max} = \sqrt{a_1^2 + a_2^2 + 2\, a_1 a_2 (1)}$$

$$A_{max} = \sqrt{(a_1 + a_2)^2} = a_1 + a_2 = 5 + 3$$

$$\mathbf{A_{max} = 8\ units}$$

- When $\phi = 180°$ (or) $\cos\phi = -1$ then resultant amplitude will be minimum.

- $$A_{min} = \sqrt{a_1^2 + a_2^2 + 2\, a_1 a_2 (-1)}$$

$$A_{min} = \sqrt{(a_1 - a_2)^2} = a_1 - a_2 = 5 - 3$$

$$\mathbf{A_{min} = 2\ units}$$

- Intensity is directly proportional to the square of the amplitude ($I \propto A^2$)

$$\frac{I_{max}}{I_{min}} = \frac{A_{max}^2}{A_{min}^2} = \frac{8^2}{2^2} = \frac{64}{4} = \frac{16}{1}$$

$$\boldsymbol{I_{max} : I_{min} = 16 : 1}$$

3. Two light sources of equal amplitudes interfere with each other. Calculate the ratio of maximum and minimum intensities.

Solution :

- Let 'a' be the amplitude ,then equation for intensity

$$I \propto 4\, a^2 cos^2 \left[\frac{\phi}{2}\right]$$

- When, $\phi = 0°$ (or) $\cos\left(\frac{\phi}{2}\right) = 1$ then, intensity will be maximum.

$$I_{max} \propto 4\, a^2$$

- When $\phi = 180°$ (or) $\cos\left(\frac{\phi}{2}\right) = 0$ then, intensity will be minimum.

$$I_{min} = 0$$

- Hence the ratio of maximum and minimum ,

$$\boldsymbol{I_{max} : I_{min} = 4\, a^2 : 0}$$

4. Two light sources have intensity of light as I0. What is the resultant intensity at a point where the two light waves have a phase difference of π/3?

Solution : $\phi = \boldsymbol{\frac{\pi}{3}}$

- Resultant intensity, $I \propto 4\, a^2 cos^2 \left[\frac{\phi}{2}\right]$ $[\because I_O \propto a^2]$

$$I = 4\, I_O\, cos^2 \left[\frac{\phi}{2}\right] = 4\, I_O\, cos^2 \left[\frac{\left(\frac{\pi}{3}\right)}{2}\right] = 4\, I_O\, cos^2 \left[\frac{\pi}{6}\right] = 4\, I_O \left[\frac{\sqrt{3}}{2}\right]^2 = 4\, I_O \left[\frac{3}{4}\right]$$

$$\boldsymbol{I = 3\, I_O}$$

5. The wavelength of a light is 450 nm. How much phase it will differ for a path of 3 mm?

Solution : $\lambda = 450\ nm = 450\ X\ 10^{-9}\ m$; $\delta = 3\ mm = 3\ X\ 10^{-3}\ m$

- Phase difference,

$$\phi = \frac{2\,\pi}{\lambda}\,\delta = \frac{2\,\pi}{450\ X\ 10^{-9}}\ X\ 3\ X\ 10^{-3} = \boldsymbol{\frac{\pi}{75}\ X 10^6\ rad = 4.19\ X\ 10^4\ rad}$$

6. In Young's double slit experiment, the two slits are 0.15 mm apart. The light source has a wavelength of 450 nm. The screen is 2 m away from the slits.

(a) Find the distance of the second bright fringe and also third dark fringe from the central maximum.

(b) Find the fringe width.

(c) How will the fringe pattern change if the screen is moved away from the slits?

(d) What will happen to the fringe width if the whole setup is immersed in waterof refractive index 4/3.

Solution : $d = 0.15\ mm = 0.15\ X\ 10^{-3}\ m$; $D = 2\ m$; $\lambda = 450\ nm = 450\ X\ 10^{-9} m$

(a) Distance of nth bright fringe from centre, $y_n = \frac{D}{d}\, n\, \lambda$

Distance of nth dark fringe centre, $y_n = \frac{D}{d}\, (2n - 1)\, \frac{\lambda}{2}$

Hencedistance of 2nd bright fringe,

$$y_2 = \frac{D}{d}\, n\, \lambda = \frac{2}{0.15\ X\ 10^{-3}}\ X\ 2\ X\ 450\ X\ 10^{-9} = 2\ X\ 2\ X\ 3000\ X\ 10^{-6}$$

$$\boldsymbol{y_2} = 12000\ X\ 10^{-6} = \mathbf{12\ X\ 10^{-3}\ m = 12\ mm}$$

And distance of 3rd dark fringe,

$$y_3 = \frac{D}{d}\frac{5\,\lambda}{2} = \frac{2}{0.15 \,X\, 10^{-3}} \,X\, \frac{5 \,X\, 450 \,X\, 10^{-9}}{2} = 5 \,X\, 3000 \,X\, 10^{-6}$$

$$\boldsymbol{y_3} = 15000 \,X\, 10^{-6} = \mathbf{15 \,X\, 10^{-3}\, m = 15\, mm}$$

(b) Fringe width,

$$\beta = \frac{\lambda\, D}{d} = \frac{450 \,X\, 10^{-9} \,X\, 2}{0.15 \,X\, 10^{-3}} = 3000 \,X\, 10^{-6} X\, 2 = \mathbf{6 \,X\, 10^{-3}\, m = 6\, mm}$$

(c) Since, $\boldsymbol{\beta \propto D}$, when distance (D) between slit and screen increases, the fringe width ($\boldsymbol{\beta}$) also increases.

(d) When, $n = \frac{4}{3}$,then fringe width (β^1) in water

$$\beta^1 = \frac{\lambda^1 D}{d} = \frac{\lambda\, D}{n\, d} = \frac{\beta}{n} = \frac{6 \,X\, 10^{-3}}{\left(\frac{4}{3}\right)} = \frac{3 \,X\, 6 \,X\, 10^{-3}}{4} = \frac{18 \,X\, 10^{-3}}{4} \qquad \left[\because\ \lambda^1 = \frac{\lambda}{n}\right]$$

$$\boldsymbol{\beta^1 = 4.5 \,X\, 10^{-3}\, m = 4.5\, mm}$$

7. Lights of two wavelengths 560 nm and 420 nm are used in Young's double slit experiment. Find the least distance from the central fringe where the bright fringes of the two wavelengths coincide.

Solution : $\boldsymbol{\lambda_1} = 560\, nm = 560 \,X\, 10^{-9}\, m$; $\boldsymbol{\lambda_2} = 420\, nm = 420 \,X\, 10^{-9}\, m$

- Here for given 'y', $\boldsymbol{\lambda \propto \frac{1}{n}}$ .Here, nth order bright fringe of longer wavelength λ_1 coincides with (n+1)th order bright fringe of shorter wavelength λ_2,
- Distance of nth bright fringe ; $y_n = \frac{D}{d}\, n\, \lambda$
- Hence, $\frac{D}{d}\, n\, \lambda_1 = \frac{D}{d}\,(n+1)\,\lambda_2$

$$n\,\lambda_1 = (n+1)\,\lambda_2$$

$$(or) \qquad \frac{(n+1)}{n} = \frac{\lambda_1}{\lambda_2}$$

$$(or) \qquad 1 + \frac{1}{n} = \frac{\lambda_1}{\lambda_2}$$

$$\frac{1}{n} = \frac{\lambda_1}{\lambda_2} - 1$$

$$\frac{1}{n} = \frac{560 \,X\, 10^{-9}}{420 \,X\, 10^{-9}} - 1$$

$$\frac{1}{n} = \frac{4}{3} - 1 = \frac{1}{3}$$

$$\therefore \qquad \boldsymbol{n = 3}$$

- Thus, the 3rd bright fringe of λ_1 and 4th bright fringe of λ_2 coincide at the least distance y.
- The least distance from the central fringe where the bright fringes of the two wavelengths coincides is

$$y_n = \frac{D}{d}\, n\, \lambda = \frac{1}{3 \,X\, 10^{-3}} \,X\, 3 \,X\, 560 \,X\, 10^{-9}$$

$$\boldsymbol{y_n} = \mathbf{560 \,X\, 10^{-6}\, m = 0.56 \,X\, 10^{-3}\, m = 0.56\, mm}$$

8. Find the minimum thickness of a film of refractive index 1.25, which will strongly reflect the light of wavelength 589 nm. Also find the minimum thickness of the film to be anti-reflecting.

Solution : $\lambda = 589\, nm = 589 \,X\, 10^{-9} m$; $\mu = 1.25$

- For the film to have strong reflection, the reflected waves should interfere constructively. Hence the path difference,

$$2\,\mu\, t = (2n-1)\frac{\lambda}{2}$$

- The least optical path difference, $n = 1$, then

$$2\,\mu\, t = \frac{\lambda}{2}$$

$$\therefore \qquad t = \frac{\lambda}{4\,\mu} = \frac{589 \,X\, 10^{-9}}{4 \,X\, 1.25} = \frac{589 \,X\, 10^{-9}}{5}$$

$$\boldsymbol{t = 117.8 \,X\, 10^{-9}\, m = 117.8\, nm}$$

- For the film to be anti-reflecting, the reflected rays should interfere destructively. Hence the path difference,

$$2\,\mu\, t = n\,\lambda$$

- The least optical path difference, $n = 1$, then

$$2\,\mu\, t = \lambda$$

$$\therefore \qquad t = \frac{\lambda}{2\,\mu} = \frac{589 \,X\, 10^{-9}}{2 \,X\, 1.25} = \frac{589 \,X\, 10^{-9}}{2.5} = \frac{5890 \,X\, 10^{-9}}{25}$$

$$\boldsymbol{t = 235.6 \,X\, 10^{-9}\, m = 235.6\, nm}$$

9. Light of wavelength 500 nm passes through a slit of 0.2 mm wide. The diffraction pattern is formed on a screen 60 cm away. Determine the,
(a) angular spread of central maximum
(b) the distance between the central maximum and the second minimum.

Solution : $\lambda = 500\, nm = 500 \,X\, 10^{-9} m$; $a = 0.2\, mm = 0.2 \,X\, 10^{-3} m$

$$D = 60\, cm = 60 \,X\, 10^{-2} m$$

(a) Equation for diffraction minimum is, $a \sin\theta = n\,\lambda$

The central maximum is spread up to the first minimum. Hence, n = 1

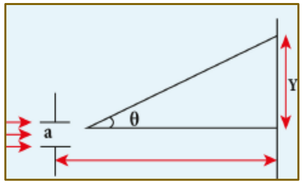
$$a \sin\theta = \lambda$$

$$(or) \qquad \sin\theta = \frac{\lambda}{a} = \frac{500 \,X\, 10^{-9}}{0.2 \,X\, 10^{-3}} = \frac{5000 \,X\, 10^{-6}}{2} = 2500 \,X 10^{-6}$$

$$(or) \qquad \sin\theta = 0.0025$$

$$(or) \qquad \boldsymbol{\theta = \sin^{-1} 0.0025 = 0.0025\, rad}$$

From the figure, for first mininmum



$$\tan\theta = \frac{y_1}{D}$$

As θ is small, $\sin\theta \approx \tan\theta$ For first minimum

$$a \sin\theta = \lambda$$

$$a\,\frac{y_1}{D} = \lambda$$

$$\therefore\ y_1 = \frac{\lambda\, D}{a} = \frac{500 \,X\, 10^{-9} \,X 60 \,X\, 10^{-2}}{0.2 \,X\, 10^{-3}} = \frac{5000 \,X\, 10^{-9} \,X\, 60 \,X\, 10^{-2}}{2 \,X\, 10^{-3}} = 150000 \,X 10^{-8}$$

$$\boldsymbol{y_1 = 1.5\, X 10^{-3}\, m = 1.5\, mm}$$

For second minimum $(n = 2)$

$$a \sin\theta = 2\lambda$$

$$a \frac{y_2}{D} = 2\lambda$$

$$\therefore \quad y_2 = \frac{2\lambda D}{a} = \frac{2 X 500 X 10^{-9} X 60 X 10^{-2}}{0.2 X 10^{-3}}$$

$$y_2 = \frac{20 X 500 X 10^{-9} X 60 X 10^{-2}}{2 X 10^{-3}} = 300000 X 10^{-8}$$

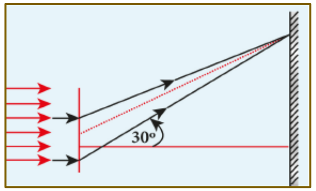
$$\boldsymbol{y_2 = 3.0 X 10^{-3} m = 3 mm}$$

(b) The distance between the central maximum and second minimum is,

$$\boldsymbol{y_2 - y_1 = 3 - 1.5 = 1.5 mm}$$

10. A monochromatic light of wavelength 5000 Å passes through a single slit producing diffraction pattern for the central maximum as shown in the figure. Determine the width of the slit.

Solution : $\lambda = 5000\ A° = 5000 X 10^{-10} m$; $\theta = 30°$; $n = 1$



- Equation for diffraction minimum is,

$$a \sin\theta = n\lambda$$

- For first minimum $(n = 1)$

$$a \sin\theta = \lambda$$

$$(or) \quad a = \frac{\lambda}{\sin\theta}$$

$$a = \frac{5000 X 10^{-10}}{\sin 30°} = \frac{5000 X 10^{-10}}{\left(\frac{1}{2}\right)}$$

$$a = 10000 X 10^{-10} = 1 X 10^{-6} m = 0.001 X 10^{-3}$$

$$\boldsymbol{a = 0.001 mm}$$

11. Calculate the distance upto which ray optics is a good approximation for light of wavelength 500 nm falls on an aperture of width 0.5 mm.

Solution : $\lambda = 500\ \mathbf{nm} = 500 X 10^{-9} m$; $a = 0.5 mm = 0.5 X 10^{-3} m$

- Fresnel distance, $z = \frac{a^2}{2\lambda}$

$$z = \frac{(0.5 X 10^{-3})^2}{2 X 500 X 10^{-9}} = \frac{0.25 X 10^{-6}}{1000 X 10^{-9}} = \frac{0.25 X 10^{-6}}{10^{-6}}$$

$$\boldsymbol{z = 0.25 m = 25 cm}$$

12. A diffraction grating consists of 4000 slits per centimeter. It is illuminated by a monochromatic light. The second order diffraction maximum is produced at an angle of 30°. What is the wavelength of the light used?

Solution : $N = 4000\ lines/cm = 400000\ lines/m$; $\theta = 30°$; $m = 2$

- Equation for diffraction maximum for grating is,

$$\sin\theta = N m \lambda$$

$$\lambda = \frac{\sin\theta}{N m} = \frac{\sin 30°}{400000 X 2} = \frac{\left(\frac{1}{2}\right)}{8 X 10^5} = \frac{1}{16} X 10^{-5}$$

$$\boldsymbol{\lambda = 0.0625 X 10^{-5} = 6250 X 10^{-10} m = 6250 A°}$$

13. A monochromatic light of wavelength of 500 nm strikes a grating and produces fourth order maximum at an angle of 30°. Find the number of slits per centimeter.

Solution : $\lambda = 500\ nm = 500 X 10^{-9} m$; $\theta = 30°$; $m = 4$

- Equation for diffraction maximum for grating is ; $\sin\theta = N m \lambda$

$$\therefore \quad N = \frac{\sin\theta}{\lambda m} = \frac{\sin 30°}{500 X 10^{-9} X 4} = \frac{\left(\frac{1}{2}\right)}{2000 X 10^{-9}} = \frac{1}{4} X 10^6 = 0.25 X 10^6$$

$$\boldsymbol{N = 2.5 X 10^5\ lines/m = 2500\ lines/cm}$$

14. The optical telescope in the Vainu Bappu observatory at Kavalur has an objective lens of diameter 2.3 m. What is its angular resolution if the wavelength of light used is 589 nm?

Solution : $\lambda = 589\ nm = 589 X 10^{-9} m$; $a = 2.3 m$

- Equation for angular resolution,

$$\theta = \frac{1.22\lambda}{a} = \frac{1.22 X 589 X 10^{-9}}{2.3}$$

$$\theta = 3.125 X 10^2 X 10^{-9} = \boldsymbol{3.125 X 10^{-7}\ radian \approx 0.0011'}$$

15. Two polaroids are kept with their transmission axes inclined at 30°. Unpolarised light of intensity I falls on the first polaroid. Find out the intensity of light emerging from the second polaroid.

Solution :

- As the intensity of the unpolarised light falling on the first polaroid is I, the intensity of polarized light emerging from it will be,

$$\boldsymbol{I_O = \frac{I}{2}}$$

- Let I^1 be the intensity of light emerging from the second polaroid.

$$I^1 = I_O \cos^2\theta$$

$$I^1 = \frac{I}{2}\cos^2 30° = \frac{I}{2}\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{I}{2} X \frac{3}{4}$$

$$\boldsymbol{I^1 = \left(\frac{3}{8}\right) I}$$

16. Two polaroids are kept crossed (transmission axes at 90°) to each other.

(a) What will be the intensity of the light coming out from the second polaroid when an unpolarised light of intensity I falls on the first polaroid?

(b) What will be the intensity of light coming out from the second polaroid if a third polaroid is kept in between at 45° inclination to both of them.

Solution :

(a) As the intensity of the unpolarised light falling on the first polaroid P_1 is I, the intensity of polarized light emerging from it will be ; $\boldsymbol{I_O = \frac{I}{2}}$

Let I^1, be the intensity of light emerging from the second polaroid P_2. From Malus' law

$$I^1 = I_O \cos^2\theta = \frac{I}{2}\cos^2 90° = 0$$

No light comes out from the second polaroid.

(b) If third polaroid P_3 kept between P_1 & P_2 at 45°, the intensity of emergent light from P_3

$$I^1 = I_O \cos^2\theta = \frac{I}{2}\cos^2 45° = \frac{I}{2}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I}{2} X \frac{1}{2} = \mathbf{\frac{I}{4}}$$

Finally, the intensity of emergent light from P_2,

$$I^{11} = I^1 \cos^2\theta = \frac{I}{4}\cos^2 45° = \frac{I}{4}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I}{4} X \frac{1}{2} = \mathbf{\frac{I}{8}}$$

17. Find the polarizing angles for (i) glass of refractive index 1.5 and (ii) water of refractive index 1.33.

Solution :

(i) When, $n = 1.5$, from Brewster's law ; $\tan i_P = n$ (or) $i_P = \tan^{-1} n$

$$\boldsymbol{i_P = \tan^{-1} 1.5 = 56.3°}$$

(ii) When, $n = 1.33$ from Brewster's law ;, $\tan i_P = n$ (or) $i_P = \tan^{-1} n$

$$\boldsymbol{i_P = \tan^{-1} 1.33 = 53.1°}$$

18. What is the angle at which a glass plate of refractive index 1.65 is to be kept with respect to the horizontal surface so that an unpolarised light travelling horizontal after reflection from the glass plate is found to be plane polarised?

Solution : $n = 1.65$

- From Brewster's law ; $\tan i_P = n$

  (or) $i_P = \tan^{-1} n = \boldsymbol{\tan^{-1} 1.65 = 58.8°}$
- The angle at which a glass plate to be kept with respect to the horizontal surface $\boldsymbol{= 90° - 58.8° = 31.2°}$

19. A man with a near point of 25 cm reads a book which has small print using a magnifying lens of focal length 5 cm.

(a) What are the closest and the farthest distances at which he should keep the lens from the book?

(b) What are the maximum and the minimum magnification possible?

Solution : $D = 25\ cm$; $f = 5\ cm$

- Closest distance of object $= u$

  Image distance (near point focusing) $= v = -25\ cm$
- Farthest distance of object $= u^1$

  Image distance (normal focusing) $= v^1 = \infty$

(a) Lens equation for near point focusing,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad (or) \quad \frac{1}{u} = \frac{1}{v} - \frac{1}{f} = \frac{1}{(-25)} - \frac{1}{5} = -\frac{1}{25} - \frac{1}{5} = \frac{-1-5}{25} = -\frac{6}{25}$$

$$\boldsymbol{u = -\frac{25}{6} = -4.167\ cm}$$

Lens equation for normal focusing,

$$\frac{1}{v^1} - \frac{1}{u^1} = \frac{1}{f^1} \ (or)\ \frac{1}{u^1} = \frac{1}{v^1} - \frac{1}{f^1} = \frac{1}{\infty} - \frac{1}{5} = -0 - \frac{1}{5} = -\frac{1}{5}$$

$$\boldsymbol{u^1 = -5\ cm}$$

The closest distance between the lens and the book is, u $= \boldsymbol{-4.167\ cm}$

The farthest distance at which the person can keep the book is $= \boldsymbol{-5cm}$

(b) Magnification in near point focusing,

$$\boldsymbol{m} = 1 + \frac{D}{f} = 1 + \frac{25}{5} = 1 + 5 = \mathbf{6}$$

Magnification in normal focusing,

$$\boldsymbol{m} = \frac{D}{f} = \frac{25}{5} = \mathbf{5}$$

20. A microscope has an objective and eyepiece of focal lengths 5 cm and 50 cm respectively with tube length 30 cm. Find the magnification of the microscope in the (a) near point and (b) normal focusing.

Solution : $f_O = 5\ cm = 5\ X\ 10^{-2}\ m$; $f_e = 50\ cm = 50\ X\ 10^{-2}\ m$

$L = 30\ cm = 30\ X\ 10^{-2}\ m$; $D = 25\ cm = 25\ X\ 10^{-2}\ m$

(a) Magnification of the microscope in near point focusing,

$$m = m_o m_e = \left[\frac{L}{f_O}\right]\left[1 + \frac{D}{f_e}\right]$$

$$m = m_o m_e = \left[\frac{30\ X\ 10^{-2}}{5\ X\ 10^{-2}}\right]\left[1 + \frac{25\ X\ 10^{-2}}{50\ X\ 10^{-2}}\right]$$

$$m = m_o m_e = [6]\left[1 + \frac{1}{2}\right] = 6\ X\ \frac{3}{2} = \frac{18}{2} = \mathbf{9}$$

(b) Magnification of the microscope in normal focusing,

$$m = m_o m_e = \left[\frac{L}{f_O}\right]\left[\frac{D}{f_e}\right]$$

$$m = m_o m_e = \left[\frac{30\ X\ 10^{-2}}{5\ X\ 10^{-2}}\right]\left[\frac{25\ X\ 10^{-2}}{50\ X\ 10^{-2}}\right]$$

$$m = m_o m_e = [6]\left[\frac{1}{2}\right] = \frac{6}{2} = \mathbf{3}$$

21. A small telescope has an objective lens of focal length 125 cm and an eyepiece of focal length 2 cm. (a) What is the magnification of the telescope? (b) What is the separation between the objective and the eyepiece?(c) What is the angular separation between two stars when viewed through this telescope if they subtend 1 ' for bare eye?

Solution : $f_O = 125\ cm = 125\ X\ 10^{-2}\ m$; $f_e = 2\ cm = 2\ X\ 10^{-2}\ m$

(a) Magnification of telescope,

$$m = \frac{f_O}{f_e} = \frac{125\ X\ 10^{-2}}{2\ X\ 10^{-2}} = \frac{125}{2}$$

$$\boldsymbol{m = 62.5}$$

(b) Equation for approximate length of telescope,

$$L = f_O + f_e$$

$$L = (125\ X\ 10^{-2}) + (2\ X\ 10^{-2}) = (125 + 2)\ X\ 10^{-2}$$

$$\boldsymbol{L = 127\ X\ 10^{-2}\ m = 1.27\ m}$$

(c) The angular separation,

$$m = \frac{\theta_i}{\theta_o} \quad (or) \quad \theta_i = m\,\theta_o$$

$$\boldsymbol{\theta_i = 62.5\ X\ 1' = 62.5' = \frac{62.5}{60} = 1.04°}$$

22. Calculate the power of the lens of the spectacles needed to rectify the defect of nearsightedness for a person who could see clearly up to a distance of 1.8 m.

Solution : $x = 1.8\, m$

- The lens should have a focal length of $\boldsymbol{f = -x = -1.8\, m}$
- It is a concave (or) diverging lens.
- The power of the lens is,

$$P = \frac{1}{f} = -\frac{1}{1.8}$$

$$\boldsymbol{P = -0.56}\ \textbf{diopter}$$

23. A person has farsightedness with the far distance he could see clearly is 75 cm. Calculate the power of the lens of the spectacles needed to rectify the defect.

Solution : $y = 75\, cm = 75\ X\ 10^{-2}\, m$

- The lens should have a focal length of f,

$$f = \frac{y\ X\ 25\, cm}{y - 25\, cm}$$

$$f = \frac{75\, cm\ X\ 25\, cm}{75\, cm - 25\, cm} = \frac{1875}{50}\, cm$$

$$\boldsymbol{f = \frac{1875}{50}\, cm = 37.5\, cm}$$

- It is a convex lens (or) converging lens.
- The power of the lens is,

$$P = \frac{1}{f} = -\frac{1}{37.5\ X\ 10^{-2}} = \frac{10^{-2}}{37.5} = \frac{100}{37.5}$$

$$\boldsymbol{P = 2.67}\ \textbf{diopter}$$

EXERCISE PROBLEMS

1. The ratio of maximum and minimum intensities in an interference pattern is 36 : 1. What is the ratio of the amplitudes of the two interfering waves?

Solution : $I_{max} : I_{min} = 36 : 1$

- We have, $\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}$ (or) $\frac{a_1+a_2}{a_1-a_2} = \sqrt{\frac{I_{max}}{I_{min}}} = \sqrt{\frac{36}{1}} = 6$
- Hence,

$$a_1 + a_2 = 6\,(a_1 - a_2)$$
$$a_1 + a_2 = 6\,a_1 - 6\,a_2$$
$$a_2 + 6\,a_2 = 6\,a_1 - a_1$$
$$7\,a_2 = 5\,a_1$$
$$\frac{a_1}{a_2} = \frac{7}{5}$$
$$\boldsymbol{a_1 : a_2 = 7 : 5}$$

2. In Young's double slit experiment, 62 fringes are seen on a screen for sodium light of wavelength 5893 Å. If violet light of wavelength 4359 Å is used in place of sodium light, how many fringes will be seen?

Solution : $\lambda_1 = 5893\ A^\circ$; $\lambda_2 = 4359\ A^\circ$; $n_1 = 62$

- Equation of fringe width ; $\beta = \frac{\lambda D}{d}$
- Hence fringe width of n-fringes , $n\,\beta = n\frac{\lambda D}{d}$ .Thus

$$n_1 \frac{\lambda_1 D}{d} = n_2 \frac{\lambda_2 D}{d}$$
$$n_1\lambda_1 = n_2\lambda_2$$
$$n_2 = \frac{n_1\lambda_1}{\lambda_2}$$
$$n_2 = \frac{62\ X\ 5893\ X\ 10^{-10}}{4359\ X\ 10^{-10}} = \frac{62\ X\ 5893}{4359}$$
$$\boldsymbol{n_2 = 83.81 \approx 84}$$

No	Log
62	1.7924
5893	3.7703
(+)	5.5627
4359	3.6394
(-)	1.9233
ALog	$8.3181\ X\ 10^1$

3. Light of wavelength 600 nm that falls on a pair of slits producing interference pattern on a screen in which the bright fringes are separated by 7.2 mm. What must be the wavelength of another light which produces bright fringes separated by 8.1 mm with the same apparatus?

Solution : $\lambda_1 = 600\, nm = 600\ X\ 10^{-9}\, m$; $\beta_1 = 7.2mm = 7.2\ X\ 10^{-3}\, m$;

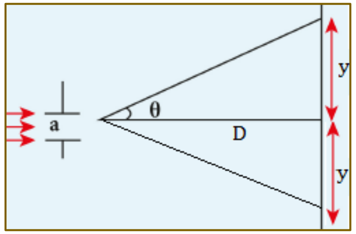
$\beta_2 = 8.1\, mm = 8.1\ X\ 10^{-3}\, m$

- Equation of fringe width ; $\beta = \frac{\lambda D}{d}$
- Hence,

$$\frac{\beta_1}{\beta_2} = \frac{\lambda_1}{\lambda_2} \quad \text{(or)} \quad \lambda_2 = \lambda_1 \frac{\beta_2}{\beta_1}$$

$$\lambda_2 = 600\ X\ 10^{-9}\ X\ \frac{8.1\ X\ 10^{-3}}{7.2\ X\ 10^{-3}} = \frac{4860}{7.2} X\ 10^{-9} = 675\ X\ 10^{-9} m = 675\, nm$$

4. A beam of light of wavelength 600 nm from a distant source falls on a single slit 1 mm wide and the resulting diffraction pattern is observed on a screen 2 m away. What is the distance between the first dark fringes on either side of the central bright fringe?



Solution : $a = 1mm = 1\ X\ 10^{-3}\ m$;

$\lambda = 600\ nm = 600\ X\ 10^{-9}\ m$; $D = 2\ m$

- Equation for diffraction minimum,

  $a\ sin\ \theta = n\ \lambda \quad (or) \quad a\ \frac{y}{D} = n\ \lambda$

- Condition for 1st minimum (n=1)

$$a\ \frac{y}{D} = \lambda$$

$$\therefore \quad y = \frac{D\ \lambda}{a} = \frac{2\ X\ 600\ X\ 10^{-9}}{1\ X\ 10^{-3}} = 1200\ X\ 10^{-6} m = 1.2\ X 10^{-3}\ m = 1.2\ mm$$

- Hence distance between the first fringe on either side of the central bright fringe

$$\boldsymbol{Y = 2\ y = 2\ X 1.2 = 2.4\ mm}$$

5. Light of wavelength of 5000 Å produces diffraction pattern of the single slit of width 2.5 μm. What is the maximum order of diffraction possible?

Solution : $a = 2.5\ \mu m = 2.5\ X\ 10^{-6}\ m$; $\lambda = 5000\ Å = 5000\ X\ 10^{-10}\ m$

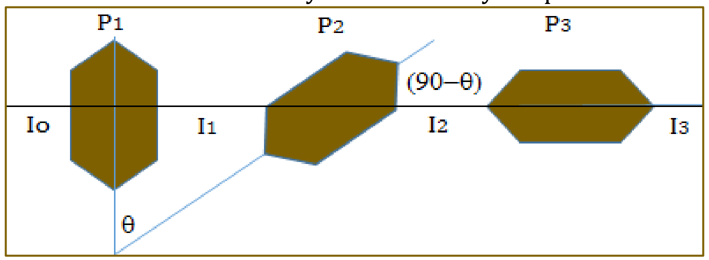
- Equation for diffraction minimum : $\boldsymbol{a\ sin\ \theta = n\ \lambda}$
- For maximum order ; $\theta = 90°$ (or) $sin\ \theta = 1$. Hence

$$\boldsymbol{n} = \frac{\boldsymbol{a\ sin\ \theta}}{\boldsymbol{\lambda}} = \frac{2.5\ X\ 10^{-6}\ X\ 1}{5000\ X\ 10^{-10}} = 0.5\ X\ 10^{1} = \boldsymbol{5}$$

6. I_0 is the intensity of light existing between two cross Polaroids kept with their axes perpendicular to each other. A third polaroid is introduced between them. What must be the angle between the axes of first and the newly introduced polaroid to get the maximum light from the whole arrangement?

Solution :

- If the intensity of the unpolarised light is *I* then the intensity of polarised light will be ***I*/2**. The other half of intensity is restricted by the polariser.



- Intensity of incident light on $P_1 \quad = I_o$
- Intensity of emergent light from P_1 ; $I_1 = \frac{I_o}{2}$
- Intensity of emergent light from P_2 ; $I_2 = I_1 \cos^2\theta = \frac{I_o}{2}\cos^2\theta$
- Intensity of emergent light from P_3 ; $I_3 = I_2 \cos^2(90-\theta)$

$$I_3 = \frac{I_o}{2}\cos^2\theta\cos^2(90-\theta)$$

$$I_3 = \frac{I_o}{2}\cos^2\theta\ \sin^2\theta = \frac{I_o}{2}\ [sin\ \theta\ \ cos\ \theta]^2 = \frac{I_o}{2}\left[\frac{sin\ 2\ \theta}{2}\right]^2$$

$$I_3 = \frac{I_o}{2}\left[\frac{\sin^2 2\theta}{4}\right] = \frac{I_o}{8}\ \sin^2 2\theta$$

- When $\sin^2 2\theta = maximum$, then I_3 will bemaximum. (i.e.)

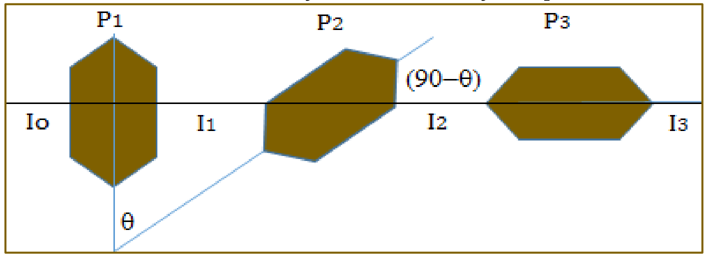
$$\sin^2 2\theta = 1 \quad (or) \quad sin\ 2\theta = 1 \quad (or) \quad 2\theta = 90°$$

$$\therefore \quad \boldsymbol{\theta = 45°}$$

7. An unpolarised light of intensity 32 Wm^{-2} passes through three Polaroids such that the axes of the first and the last Polaroids are at 90°. What is the angle between the axes of the first and middle Polaroids so that the emerging light has an intensity of only 3 Wm^{-2}?

Solution : : $I_o = 32\ Wm^{-2}$; $I_3 = 3\ Wm^{-2}$

- If the intensity of the unpolarised light is *I* then the intensity of polarised light will be ***I*/2**. The other half of intensity is restricted by the polariser.



- Intensity of incident light on $P_1 \quad = I_o$
- Intensity of emergent light from P_1 ; $I_1 = \frac{I_o}{2}$
- Intensity of emergent light from P_2 ; $I_2 = I_1 \cos^2\theta = \frac{I_o}{2}\cos^2\theta$
- Intensity of emergent light from P_3 ; $I_3 = I_2 \cos^2(90-\theta)$

$$I_3 = \frac{I_o}{2}\cos^2\theta\cos^2(90-\theta)$$

$$I_3 = \frac{I_o}{2}\cos^2\theta\ \sin^2\theta = \frac{I_o}{2}\ [sin\ \theta\ \ cos\ \theta]^2 = \frac{I_o}{2}\left[\frac{sin\ 2\ \theta}{2}\right]^2$$

$$I_3 = \frac{I_o}{2}\left[\frac{\sin^2 2\theta}{4}\right] = \frac{I_o}{8}\ \sin^2 2\theta$$

$$3 = \frac{32}{8}\ \sin^2 2\theta$$

$$(or) \quad \sin^2 2\theta = \frac{24}{32} = \frac{3}{4}$$

$$(or) \quad sin\ 2\theta = \frac{\sqrt{3}}{2}$$

$$(or) \quad 2\theta = \sin^{-1}\left[\frac{\sqrt{3}}{2}\right] = 60°$$

$\therefore \qquad \boldsymbol{\theta = 30°}$

8. The reflected light is found to be plane polarised when an unpolarized light falls on a denser medium at 60° with the normal. Find the angle of refraction and critical angle of incidence for total internal reflection in the denser to rarer medium reflection.

Solution :

- The angle of incidence at which the reflected ray get completely plane polarized is called angle of polarization (i_P). Hence $\boldsymbol{i_P = 60°}$
- At polarizing angle, the angle of refraction ,

$$\mathbf{r} = 90° - i_P = 90° - 60° = \mathbf{30°}$$

- From Brewster's law, $n = \tan i_P = \tan 60° = \sqrt{3}$
- Let i_C be the critical angle, then

$$\sin i_C = \frac{1}{n} = \frac{1}{\sqrt{3}} = 0.5774$$

$$\boldsymbol{i_C} = \sin^{-1}(0.5774) = \mathbf{35.26°} = \mathbf{35°16^I}$$

9. The near point and the far point for a person are 50 cm and 500 cm, respectively. Calculate the power of the lens the person should wear to read a book held in hand at 25 cm. What maximum distance is clearly visible for the person with this lens on the eye?

Solution : $u = -25\ cm = -0.25\ m$; $v_n = -50cm = -0.5\ m$; $v_f = -500cm = -5m$

- Power of lens ;

$$\boldsymbol{P} = \frac{1}{f} = \frac{1}{v_n} - \frac{1}{u}$$

$$\boldsymbol{P} = \frac{1}{(-0.5)} - \frac{1}{(-0.25)} = -\frac{1}{0.5} + \frac{1}{0.25} = -2 + 4 = \mathbf{2\ D}$$

- Let u_{max} be the maximum distance, then

$$\boldsymbol{P} = \frac{1}{f} = \frac{1}{v_f} - \frac{1}{u_{max}}$$

$$\mathbf{2} = \frac{1}{(-5)} - \frac{1}{u_{max}} = -\frac{1}{5} - \frac{1}{u_{max}}$$

$$\frac{1}{u_{max}} = -\frac{1}{5} - 2 = -\frac{11}{5}$$

$$u_{max} = -\frac{5}{11} = -\ 0.454\ m = -\ 45.45\ cm$$

- The maximum distance that clearly visible for the person with this lens on the eye will be $= 45.45\ cm$

10. A compound microscope has a magnifying power of 100 when the image is formed at infinity. The objective has a focal length of 0.5 cm and the tube length is 6.5 cm. What is the focal length of the eyepiece.

Solution : $m = 100$; $f_o = 0.5\ cm$; $L = 6.5\ cm$; $D = 25cm$

- When the image is formed at infinity, the real image produced by objective lens is formed at the focus of the eyepiece, then $v_o + f_e = L$

$$v_o + f_e = 6.5 \qquad ------ \quad (1)$$

- Magnification of compound microscope ;

$$m = \frac{v_o}{u_o} X \frac{D}{f_e} = -\left[1 - \frac{v_o}{f_o}\right]\frac{D}{f_e} = -\left[1 - \frac{v_o}{f_o}\right]\frac{25}{f_e}$$

$$100 = -\left[1 - \frac{v_o}{0.5}\right]\frac{25}{f_e}$$

$$4 = -\left[1 - \frac{v_o}{0.5}\right]\frac{1}{f_e}$$

$$4\ f_e = -1 + \frac{v_o}{0.5} = -1 + 2\ v_o$$

$$(or) \qquad 2\ v_o - 4\ f_e = 1 \qquad ------ \quad (2)$$

$$(1) X\ 2 \Rightarrow \qquad 2\ v_o + 2\ f_e = 13 \qquad ------ \quad (3)$$

$$(3) - (2) \Rightarrow \qquad 6\ f_e = 12$$

$$\boldsymbol{f_e} = \frac{12}{6} = \mathbf{2\ cm}$$