# HIGHER SECONDARY
# SECOND YEAR


# PHYSICS

# UNIT -6
# RAY OPTICS

## PROBLEMS AND SOLUTIONS

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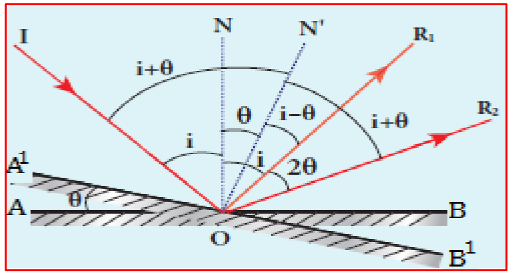
## EXAMPLE PROBLEMS

**1. Prove that for the same incident light when a reflecting surface is tilted by an angle $\theta$, the reflected light will be tilted by an angle $2\theta$.**

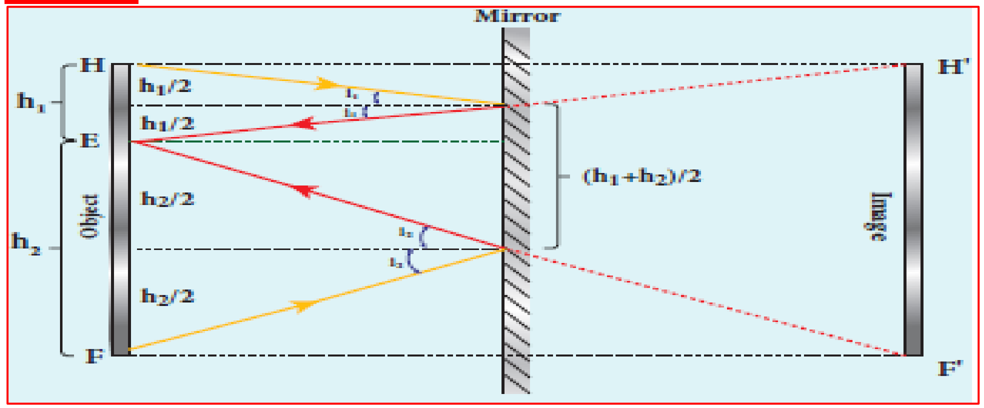
**Solution :**




- $AB$ – reflecting surface
  $IO$ – incident ray
  $OR_1$ – reflected ray
  $ON$ – normal
  $\angle ION$ – angle of incidence ($i$)
  $\angle NOR_1$ – angle of reflection (r)
- From law of refraction ;
  $$\angle ION = \angle NOR_1 = i$$
- When the surface $AB$ is tilted to $A^1B^1$ by an angle $\theta$, the normal $N$ is also is tilted to $ON^1$by the same angle $\theta$
- Now, in the tilted system,
  the angle of incidence ; $\angle ION^1 = i + \theta$
  the angle of reflection ; $\angle N^1OR_2 = i + \theta$
- The angle between $ON^1$ and $OR_1$ is ; $\angle N^1OR_1$
- The angle tilted on the reflected light is the angle between $OR1$ and $OR2$ which is,
  $$\angle R_1OR_2 = \angle N^1OR_2 - \angle N^1OR_1$$
  $$\angle R_1OR_2 = (i+\theta) - (i-\theta) = i + \theta - i + \theta$$
  $$\boldsymbol{\angle R_1OR_2 = 2\,\theta}$$

**2. What is the height of the mirror needed for a person to see his/her image fully on the mirror?**

**Solution :**




- Let us assume a person of height $h$ is standing in front of a vertical plane mirror.
- The person could see his/her head when light from the head falls on the mirror and gets reflected to the eyes. Same way, light from the feet falls on the mirror and gets reflected to the eyes.
- Let the distance between his head $H$ and eye $E$ is $h_1$ and distance between his feet $F$ and eye $E$ is $h_2$. The person's total height is ; $\boldsymbol{h = h_1 + h_2}$
- By the law of reflection, the angle of incidence and angle of reflection are the same for the two extreme reflections. The normals are now the bisectors of the angles between the incident and the reflected rays at the two points.
- By geometry, the height of the mirror needed is only half of the height of the person. (i.e.)
  $$\frac{h_1}{2} + \frac{h_2}{2} = \frac{h_1+h_2}{2} = \frac{\boldsymbol{h}}{\boldsymbol{2}}$$

**3. An object is placed at a distance of 20.0 cm from a concave mirror of focal length 15.0 cm. (a) What distance from the mirror a screen should be placed to get a sharp image? (b) What is the nature of the image?**

**Solution :** $f = -15\ cm$ ; $u = -20\ cm$ ; $v = ?$

**(a)** From the mirror equation,
$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$
$$(or) \quad \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{u-f}{f\,u}$$
$$(or) \quad v = \frac{f\,u}{u-f}$$
$$\therefore \quad v = \frac{(-15)(-20)}{(-20) - (-15)} = \frac{300}{-20+15} = \frac{300}{-5}$$
$$\boldsymbol{v = -60\ cm}$$

- The screen is to be placed at distance 60.0 cm to the left of the concave mirror.

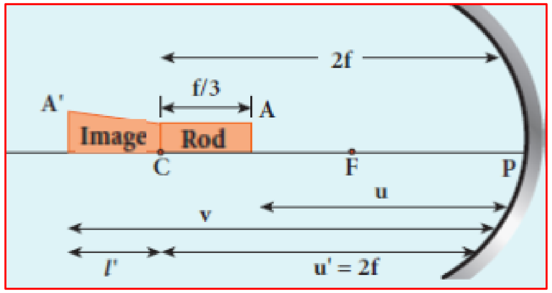
**(b)** Magnification,
$$m = \frac{h^1}{h} = -\frac{v}{u}$$
$$\boldsymbol{m} = -\frac{(-60)}{(-20)} = \boldsymbol{-3}$$

- As the sign of magnification is negative, the image is inverted.
- As the magnitude of magnification is 3, the image is enlarged three times.
- As the image is formed to the left of the concave mirror, the image is real.

**4. A thin rod of length $f/3$ is placed along the optical axis of a concave mirror of focal length $f$ such that one end of image which is real and elongated just touches the respective end of the rod. Calculate the longitudinal magnification.**

**Solution :** Object length $= l = \frac{f}{3}$ ; image length $= l^1$

- By definition,
  $$Longitudinal\ magnification = \frac{Image\ length}{object\ length}$$
  $$m = \frac{l^1}{l} = \frac{l^1}{\frac{f}{3}} = \frac{3\,l^1}{f}$$
  $$(or) \quad l^1 = \frac{m\,f}{3} \quad \text{---------} \quad (1)$$
- Image of one end coincides with the respective end of object. Thus, the coinciding end must be at centre of curvature. Thus, $u^1 = R = 2\,f$




- From figure,

$$u^1 = u + \frac{f}{3}$$

$$2f = u + \frac{f}{3}$$

(or) $$u = 2f - \frac{f}{3} = \frac{5f}{3}$$

& $$v = u + \frac{f}{3} + l^1$$

$$v = \frac{5f}{3} + \frac{f}{3} + \frac{mf}{3} = \frac{5f + f + mf}{3} = \frac{f}{3}(5 + 1 + m) = \frac{f}{3}(6 + m)$$

- From mirror ; $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$
- For concave mirror ; $\frac{1}{[-f]} = \frac{1}{\left[-\frac{5f}{3}\right]} + \frac{1}{\left[-\frac{f}{3}(6+m)\right]}$

(or) $$1 = \frac{3}{5} + \frac{3}{(6+m)}$$

(or) $$1 - \frac{3}{5} = \frac{3}{6+m}$$

(or) $$\frac{2}{5} = \frac{3}{6+m}$$

$$12 + 2m = 15$$

$$2m = 15 - 12 = 3$$

$$\boldsymbol{m = \frac{3}{2} = 1.5}$$

**5. Pure water has refractive index 1.33. What is the speed of light through it?**

**Solution :** $n = 1.33$ ; $v = ?$

$$n = \frac{c}{v}$$

(or) $$v = \frac{c}{n} = \frac{3 X 10^8}{1.33} = \frac{3 X 10^8}{\left(\frac{4}{3}\right)} = \frac{9 X 10^8}{4}$$

$$\boldsymbol{v = 2.25 X 10^8\ m\ s^{-1}}$$

- Light travels with a speed of 2.26 × 108 m s-1 through pure water.

**6. Light travels from air into a glass slab of thickness 50 cm and refractive index 1.5.**
**(a) What is the speed of light in the glass slab?**
**(b) What is the time taken by the light to travel through the glass slab?**
**(c) What is the optical path of the glass slab?**

**Solution :** $d = 50\ cm = 50 X 10^{-2} m$ ; $n = 1.5$ ; $v = ?$ ; $t = ?$ ; $d^1 = ?$

(a) Refractive index of the medium ; $n = \frac{c}{v}$

∴ Speed of light in the glass slab is

$$v = \frac{c}{n} = \frac{3 X 10^8}{1.5} = \boldsymbol{2 X 10^8\ m\ s^{-1}}$$

(b) Let 't' be the time taken by light to travel through the glass slab,

$$t = \frac{d}{v} = \frac{50 X 10^{-2}}{2 X 10^8} = 25 X 10^{-10} s = \boldsymbol{2.5 X 10^{-9}\ s}$$

(c) Optical path ; $d^1 = n\,d = 1.5 X 50 X 10^{-2} = \boldsymbol{75 X 10^{-2}\ m = 75\ cm}$

- Light would have travelled an additional 25 cm (75 cm – 50 cm) in vacuum at the same time had there been no glass slab in its path.

**7. Light travelling through transparent oil enters in to glass of refractive index 1.5. If the refractive index of glass with respect to the oil is 1.25, what is the refractive index of the oil?**

**Solution :** $n_g = 1.5$ ; $n_{go} = 1.25$ ; $n_O = ?$

- Refractive index of glass with respect to oil ; $n_{go} = \frac{n_g}{n_O}$

(or) $$n_O = \frac{n_g}{n_{go}} = \frac{1.5}{1.25} = \frac{150}{125} = \boldsymbol{1.2}$$

**8. A coin is at the bottom of a trough containing three immiscible liquids of refractive indices 1.3, 1.4 and 1.5 poured one above the other of heights 30 cm, 16 cm, and 20 cm respectively. What is the apparent depth at which the coin appears to be when seen from air medium outside? In which medium the coin will appear?**

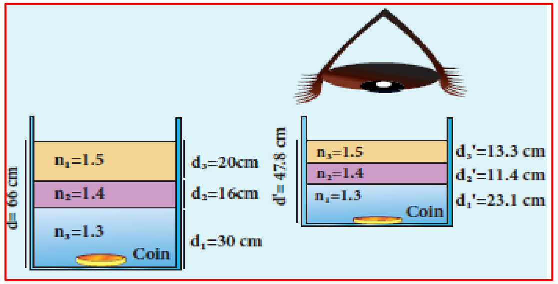
**Solution :** $d_1 = 30\ cm$ ; $d_2 = 16\ cm$ : $d_3 = 20\ cm$ ; $n_1 = 1.3$ ; $n_2 = 1.4$ : $n_3 = 1.5$

- The equations for apparent depth for each medium is,,

$$d_1^1 = \frac{d_1}{n_1} = \frac{30}{1.3} = 23.1\ cm$$

$$d_2^1 = \frac{d_2}{n_2} = \frac{16}{1.4} = 11.4\ cm$$

$$d_3^1 = \frac{d_3}{n_3} = \frac{20}{1.5} = 13.3\ cm$$




- Total depth of three medium,,

$$d = d_1 + d_2 + d_3$$

$$d = 30 + 16 + 20 = 66\ cm$$

- Total apparent depth of three medium,

$$d^1 = d_1^1 + d_2^1 + d_3^1 = 23.1 + 11.4 + 13.3 = \boldsymbol{47.8\ cm}$$

**9. What is the radius of the illumination when seen above from inside a swimming pool from a depth of 10 m on a sunny day? What is the total angle of view? [Given, refractive index of water is 4/3]**

**Solution :** $n = \frac{4}{3}$ ; $d = 10\ m$

(a) Radius of illumination,

$$R = \frac{d}{\sqrt{n^2 - 1}} = \frac{10}{\sqrt{\left(\frac{4}{3}\right)^2 - 1}} = \frac{10}{\sqrt{\frac{16}{9} - 1}}$$

$$R = \frac{10}{\sqrt{\frac{16-9}{9}}} = \frac{10 X 3}{\sqrt{16 - 9}} = \frac{30}{\sqrt{7}}$$

| No | Log |
|---|---|
| 30<br>$\sqrt{7}$ | 1. 4771<br>0. 8451 X 1/2<br>= 0.4225 |
| Nr<br>Dr | 1. 4771<br>0. 4225 |
| (-)<br>ALog | 1. 0546<br>$1.134 X 10^1$ |

$$\boldsymbol{R = 11.34\ m}$$

(b) Critical angle,

$$i_C = \sin^{-1}\left[\frac{1}{n}\right] = \sin^{-1}\left[\frac{1}{4/3}\right] = \sin^{-1}\left[\frac{3}{4}\right] = \sin^{-1}[0.75] = \mathbf{48.6}°$$

(c) The total angle of view of the cone $= 2\, i_C = 2 \, X \, 48.6° = \mathbf{97.2}°$

**10. A optical fibre is made up of a core material with refractive index 1.68 and a cladding material of refractive index 1.44. What is the acceptance angle of the fibre if it is kept in air medium without any cladding?**

**Solution :** $n_1 = 1.68$ ; $n_2 = 1.44$

- If there is cladding, then acceptance angle ;

$$i_a = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$
$$i_a = \sin^{-1}\sqrt{1.68^2 - 1.44^2}$$
$$i_a = \sin^{-1}\sqrt{2.8224 - 2.0736}$$
$$i_a = \sin^{-1}\sqrt{0.7488}$$
$$i_a = \sin^{-1}(0.8653)$$
$$\boldsymbol{i_a \approx 60°}$$

- If there is no cladding then, $n_2 = 1$. Then acceptance angle

$$i_a = \sin^{-1}\sqrt{n_1^2 - 1}$$
$$i_a = \sin^{-1}\sqrt{1.68^2 - 1}$$
$$i_a = \sin^{-1}\sqrt{2.8224 - 1} = \sin^{-1}\sqrt{1.8224}$$
$$i_a = \sin^{-1}(1.349)$$

Here $\sin^{-1}(> 1)$ is not possible. But, this includes the range 0° to 90°. Hence, all the rays entering the core from flat surface will undergo total internal reflection.

**11. The thickness of a glass slab is 0.25 m. It has a refractive index of 1.5. A ray of light is incident on the surface of the slab at an angle of 60°. Find the lateral displacement of the light when it emerges from the other side of the glass slab.**

**Solution :** $t = 0.25\, m$ ; $n = 1.5$ ; $i = 60°$

- By Snell's law ,

$$n = \frac{\sin i}{\sin r} \quad (or) \quad \sin r = \frac{\sin i}{n} = \frac{\sin 60°}{1.5} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{1.5} = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}} = 0.58$$
$$r = \sin^{-1}(0.58) = 35.25°$$

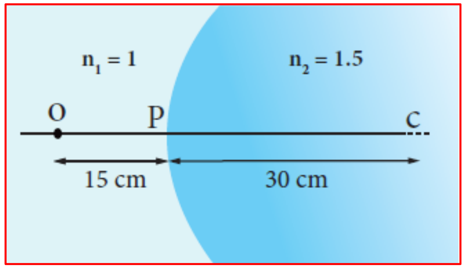
- Hence the lateral displacement,

$$L = t\left[\frac{\sin(i - r)}{\cos r}\right]$$
$$L = 0.25\, X\left[\frac{\sin(60° - 35.25°)}{\cos 35.25°}\right] = 0.25\, X\left[\frac{\sin 24.75°}{\cos 35.25°}\right]$$
$$L = 0.25\, X\left[\frac{0.4187}{0.8166}\right]$$
$$\boldsymbol{L = 0.1282\, m = 12.82\, cm}$$

| No | Log |
|---|---|
| 0.25 | $\bar{1}.3979$ |
| 0.4187 | $\bar{1}.6219$ |
| (+) | $\bar{1}.0198$ |
| 0.8166 | $\bar{1}.9120$ |
| (-) | $\bar{1}.1078$ |
| ALog | $1.282 \, X \, 10^{-1}$ |

**12. Locate the image of the point object O in the situation shown. The point C denotes the centre of curvature of the separating surface.**

**Solution :** $n_1 = 1$ ; $n_2 = 1.5$ $u = -15\, cm$ ; $R = 30\, cm$




- Equation for single spherical surface is

$$\frac{n_2}{v} - \frac{n_1}{u} = \frac{n_2 - n_1}{R}$$
$$\frac{1.5}{v} - \frac{1}{-15} = \frac{1.5 - 1}{30}$$
$$\frac{1.5}{v} + \frac{1}{15} = \frac{0.5}{30}$$
$$\frac{1.5}{v} = \frac{0.5}{30} - \frac{1}{15} = \frac{0.5 - 2}{30} = -\frac{1.5}{30}$$
$$\frac{1}{v} = -\frac{1}{30} \qquad (or) \quad \boldsymbol{v = -30\, cm}$$

- The image is a virtual image formed 30 cm to the left of the spherical surface.

**13. A biconvex lens has radii of curvature 20 cm and 15 cm for the two curved surfaces. The refractive index of the material of the lens is 1.5.**
**(a) What is its focal length?**
**(b) Will the focal length change if the lens is flipped by the side?**

**Solution :** $R_1 = 20\, cm$ ; $R_2 = -15\, cm$ ; $n = 1.5$

- From lens makers formula,

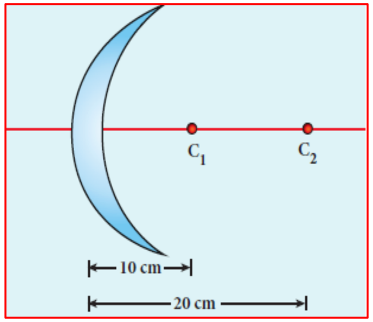
$$\frac{1}{f} = (n - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$
$$\frac{1}{f} = (1.5 - 1)\left[\frac{1}{20} - \frac{1}{-15}\right]$$
$$\frac{1}{f} = 0.5\, X\left[\frac{1}{20} + \frac{1}{15}\right] = 0.5\, X\left[\frac{3 + 4}{60}\right] = 0.5\, X\left[\frac{7}{60}\right] = \frac{7}{120}$$
$$\therefore \quad \boldsymbol{f = \frac{120}{7} = 17.14\, cm}$$

- As the focal length is positive the lens is a converging lens.
- When the lens is flipped by the side; $R_1 = 15\, cm, R_2 = -20\, cm$ ; $n = 1.5$

$$\frac{1}{f} = (n - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$
$$\frac{1}{f} = (1.5 - 1)\left[\frac{1}{15} - \frac{1}{-20}\right] = 0.5\, X\left[\frac{1}{20} + \frac{1}{15}\right]$$
$$\therefore \quad \boldsymbol{f = \frac{120}{7} = 17.14\, cm}$$

- Thus, it is concluded that the focal length of the lens will not change if it is flipped by the side. This is true for any lens.
- The **focal length is positive the lens is a converging lens.**

**14. Determine the focal length of the lens made up of a material of refractive index 1.52 as shown in the diagram. (Points $C_1$ and $C_2$ are the centers of curvature of the first and second surfaces respectively.)**




**Solution :** $n = 1.52$ ; $R_1 = 10\ cm$ ; $R_2 = 20\ cm$

- From lens makers formula,

$$\frac{1}{f} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f} = (1.52-1)\left[\frac{1}{10} - \frac{1}{20}\right]$$

$$\frac{1}{f} = 0.52\ X\left[\frac{2-1}{20}\right] = 0.52\ X\ \frac{1}{20}$$

$$\therefore\quad \boldsymbol{f = \frac{20}{0.52} = 38.46\ cm}$$

| No | Log |
|---|---|
| 20 | 1.3010 |
| 0.52 | $\bar{1}.7160$ |
| (-) | 1.5850 |
| ALog | $3.846 \times 10^{1}$ |

- As the focal length is positive, the lens is a converging lens

**15. If the focal length is 150 cm for a lens, what is the power of the lens?**

**Solution :** $f = 150\ cm = 1.5\ m$

- Power of the lens,

$$P = \frac{1}{f} = \frac{1}{1.5} = \frac{10}{15} = \boldsymbol{0.67\ diopter}$$

- As the power is positive, it is a converging lens.

**16. What is the focal length of the combination if the lenses of focal lengths –70 cm and 150 cm are in contact? What is the power of the combination?**

**Solution :** $f_1 = -70\ cm$ ; $f_2 = 150\ cm$

- The focal length of the combination lens,

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}$$

$$\frac{1}{F} = \frac{1}{-70} + \frac{1}{150} = -\frac{1}{70} + \frac{1}{150}$$

$$\frac{1}{F} = \frac{-150+70}{10500} = -\frac{80}{10500} = -\frac{8}{1050}$$

$$\therefore\quad F = -\frac{1050}{8}$$

$$\boldsymbol{F = -131.25\ cm = -1.3125\ m}$$

- As the final focal length is negative, the combination of two lenses is a diverging system of lenses.
- The power of the combination ,

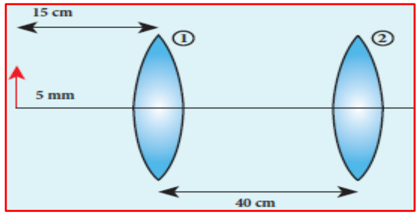
$$P = \frac{1}{f}$$

$$P = \frac{1}{-1.3125}$$

$$\boldsymbol{P = -0.7616\ D}$$

| No | Log |
|---|---|
| 1 | 0.0000 |
| 1.3125 | 0.1183 |
| (-) | $\bar{1}.8817$ |
| ALog | $7.616 \times 10^{-1}$ |

**17. An object of 5 mm height is placed at a distance of 15 cm from a convex lens of focal length 10 cm. A second lens of focal length 5 cm is placed 40 cm from the first lens and 55 cm from the object. Find (a) the position of the final image, (b) its nature and (c) its size.**




**Solution :** $h_1 = 5\ mm = 0.5\ cm$ ; $u_1 = -15\ cm$ ; $f_1 = 10\ cm$ ; $f_2 = 5\ cm$ ; $d = 40\ cm$

- For the first lens, the lens equation is,

$$\frac{1}{v_1} - \frac{1}{u_1} = \frac{1}{f_1}$$

$$(or)\quad \frac{1}{v_1} = \frac{1}{f_1} + \frac{1}{u_1} = \frac{1}{10} + \frac{1}{-15} = \frac{1}{10} - \frac{1}{15}$$

$$\frac{1}{v_1} = \frac{15-10}{150} = \frac{5}{150} = = \frac{1}{30}$$

$$\therefore\quad \boldsymbol{v_1 = 30\ cm}$$

- Equation for magnification of first lens,

$$m = \frac{h_2}{h_1} = \frac{v_1}{u_1}$$

$$\therefore\quad h_2 = h_1\frac{v_1}{u_1} = 0.5\ X\ \frac{30}{-15} = -\frac{15}{15}$$

$$\boldsymbol{h_2 = -1\ cm}$$

- As the height of the image is negative, the image is inverted and real.
- This image acts as object for second lens. The object distance for second lenses $u_2 = -(40-30) = -10\ cm$. For the second lens, the lens equation is

$$\frac{1}{v_2} - \frac{1}{u_2} = \frac{1}{f_2}$$

$$(or)\quad \frac{1}{v_2} = \frac{1}{f_2} + \frac{1}{u_2} = \frac{1}{5} + \frac{1}{-10} = \frac{1}{5} - \frac{1}{10}$$

$$\frac{1}{v_2} = \frac{10-5}{50} = \frac{5}{50} = \frac{1}{10}$$

$$\therefore\quad \boldsymbol{v_2 = 10\ cm}$$

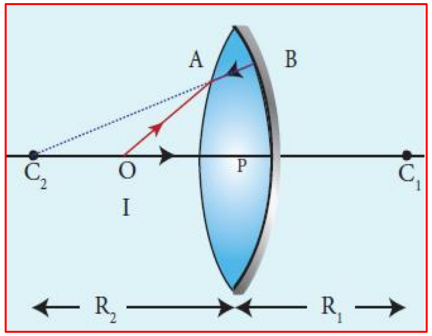
- Let the height of the final image formed by the second lens is $h_2^1$ and we have height of the object for the second lens is $h_2$. Then Equation for magnification $m^1$ for the second lens is,

$$m^1 = \frac{h_2^1}{h_2} = \frac{v_2}{u_2}$$

$$\boldsymbol{h_2^1} = h_2\frac{v_2}{u_2} = (-1)\ X\ \frac{10}{(-10)} = \boldsymbol{1\ cm = 10\ mm}$$

(a) Thus the final image is formed ***10 cm to the right of the second lens***.
(b) As the height of the image is positive, the image is ***erect and real***.
(c) The size (i.e.) height of the final image is***10 mm***

**18. A thin biconvex lens is made up of a glass of refractive index 1.5. The two surfaces have equal radii of curvature of 30 cm each. One of its surfaces is made reflecting by silvering it from outside. (a) What is the focal length and power of this silvered lens? (b) Where should an object be placed in front of this lens so that the image is formed on the object itself?**




**Solution :** $n = 1.5$; $R_1 = 30$ cm; $R_2 = -30$ cm;

(a) By Lens makers formula,focal length of lens ;

$$\frac{1}{f_l} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\frac{1}{f_l} = (1.5-1)\left[\frac{1}{30} - \frac{1}{(-30)}\right] = (0.5)\left[\frac{2}{30}\right] = \frac{1}{2}\left[\frac{2}{30}\right] = \frac{1}{30}$$

$$\therefore \quad \boldsymbol{f_l = 30\ cm = 0.3\ m}$$

- And focal length of mirror ;

$$\boldsymbol{f_m} = \frac{R_2}{2} = \frac{-30}{2} = -15\ cm = \boldsymbol{-0.15\ m}$$

- Now the focal length of the silvered lens is,

$$\frac{1}{-f} = \left[\frac{2}{f_l} + \frac{1}{-f_m}\right] = \left[\frac{2}{30} + \frac{1}{-(-15)}\right] = \left[\frac{2}{30} + \frac{1}{15}\right] = \frac{4}{30} = \frac{2}{15} = \frac{2}{7.5}$$

$$\therefore \quad \boldsymbol{f = -7.5\ cm = -0.075\ m}$$

- The silvered mirror behaves as a concave mirror with its focal length on left side.
- The power of the silvered lens,

$$P = 2P_1 + P_m$$

$$P = \frac{2}{f_l} + \frac{1}{-f_m} = \frac{4}{30\ X\ 10^{-2}} = \frac{400}{30} = \frac{40}{3} = 13.33D$$

- As the power is positive it is a converging system.

**Note:**

- Here, we come across a silvered lens which has negative focal length and positive power. Which implies that the focal length is to the left and the system is a converging one. Such situations are possible in silvered lenses because a silvered lens is basically a modified mirror.

(b) Here both $u$ and $v$ are sane $(v = u)$ as the image coincides with the object.From the mirror formula ;

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} = \frac{1}{u} + \frac{1}{u} = \frac{2}{u}$$

$$(or) \quad u = 2f = 2(-7.5) = -15\ cm = -0.15m$$

- The object is to be placed to the left of the silvered lens.

**19. A monochromatic light is incident on an equilateral prism at an angle 30° and is emergent at an angle of 75°. What is the angle of deviation produced by the prism?**

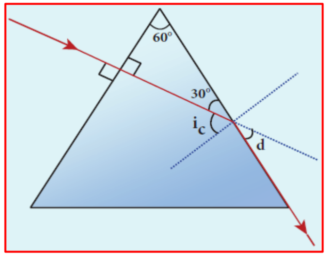
**Solution :** $A = 60°$ ; $i_1 = 30°$ ; $i_2 = 75°$

- Equation for angle of deviation,

$$d = i_1 + i_2 - A$$

$$d = (30° + 75°) - 60° = 105° - 60° = \boldsymbol{45°}$$

**20. Light ray falls at normal incidence on the first face and emerges gracing the second face for an equilateral prism.**
**(a) What is the angle of deviation produced?**
**(b) What is the refractive index of the material of the prism?**




**Solution :** $A = 60°$ ; $i_1 = 0°$ ; $i_2 = 90°$

(a) Equation for angle of deviation,

$$d = i_1 + i_2 - A$$

$$d = (0° + 90°) - 60° = 90° - 60°$$

$$\boldsymbol{d = 30°}$$

(b) The light inside the prism must be falling on the second face at critical angle as it graces the boundary. $i_c = 90° - 30° = 60°$

- Critical angle and refractive index are related as

$$n = \frac{1}{\sin i_C} = \frac{1}{\sin 60°}$$

$$n = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{2}{\sqrt{3}} = 2\ X\ 0.577 \qquad \left[\because \frac{1}{\sqrt{3}} = 0.577\right]$$

$$\boldsymbol{n = 1.154}$$

**21. The angle of minimum deviation for an equilateral prism is 37°. Find the refractive index of the material of the prism.**

**Solution :** $A = 60°$ ; $D = 37°$

- Equation for refractive index is

$$n = \frac{\sin\left(\frac{A+D}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$n = \frac{\sin\left(\frac{60° + 37°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin\left(\frac{97°}{2}\right)}{\sin\left(\frac{60°}{2}\right)} = \frac{\sin(48.5°)}{\sin(30°)} = \frac{0.75}{0.5} = \frac{75}{5}$$

$$\boldsymbol{n = 1.5}$$

**22. Find the dispersive power of a prism if the refractive indices of flint glass for red, green and violet colours are 1.613, 1.620 and 1.632 respectively.**

**Solution :** $n_V = 1.632$ ; $n_G = 1.620$ ; $n_R = 1.613$

- The dispersive power

$$\omega = \frac{n_V - n_R}{n_G - 1} = \frac{1.632 - 1.613}{1.620 - 1} = \frac{0.019}{0.620} = \boldsymbol{0.03065}$$

## EXERCISE PROBLEMS

**1. An object of 4 cm height is placed at 6 cm in front of a concave mirror of radius of curvature 24 cm. Find the position, height, magnification and nature of the image.**

**Solution :** $h = 4\ cm$ ; $R = -24\ cm$ ; $u = -6\ cm$

**(i) <u>Position of the image:</u>**

- From the relation between focal length (f) and radius of curvature (R),

$$R = 2f \qquad (or) \qquad f = \frac{R}{2} = \frac{-24}{2} = -12\ cm$$

- From mirror equation ; $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$$\therefore \frac{1}{v} = \frac{1}{f} - \frac{1}{u} = \frac{1}{(-12)} - \frac{1}{(-6)} = -\frac{1}{12} + \frac{1}{6} = \frac{-1+2}{12} = \frac{1}{12}$$

$(or)$ $\boldsymbol{v = +12\ cm}$

**(ii) <u>Magnificantion :</u>**

- Magnification is given by ; $\boldsymbol{m} = -\frac{v}{u} = -\frac{12}{(-6)} = \boldsymbol{+2}$

**(iii) <u>Height of the image:</u>**

- Magnification; $\boldsymbol{m} = \frac{\boldsymbol{h^1}}{\boldsymbol{h}}$ Hence height of the image ; $\boldsymbol{h^1} = m\,h = 2\ X\ 4 = \boldsymbol{8\ cm}$
- ***Thus the imageis erect, virtual, twice the height of object formed on right side of mirror***

**2. An object is placed in front of a concave mirror of focal length 20 cm. The image formed is three times the size of the object. Calculate two possible distances of the object from the mirror.**

**Solution :** $f = -20\ cm = -20\ X\ 10^{-2} m$

- From the equation of magnification,

$$m = \frac{f}{f-u} \qquad (or) \qquad u = f - \frac{f}{m}$$

- For real image, = $-3$ . Hence the distance of the object

$$u = (-20) - \frac{(-20)}{(-3)} = -20 - \frac{20}{3} = \frac{-60-20}{3}$$

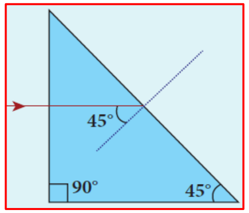
$$\boldsymbol{u = -\frac{80}{3}\ cm}$$

- For virtual image, = $+3$ . Hence the distance of the object

$$u = (-20) - \frac{(-20)}{3} = -20 + \frac{20}{3} = \frac{-60+20}{3}$$

$$\boldsymbol{u = -\frac{40}{3}\ cm}$$

**3. A beam of light consisting of red, green and blue is incident on a right-angled prism as shown in figure. The refractive index of the material of the prism for the above red, green and blue colours are 1.39, 1.44 and 1.47 respectively. What are the colours suffer total internal reflection?**




**Solution :** : $\boldsymbol{i} = 45°$; $n_R = 1.39$ ; $n_G = 1.44$ ; $n_B = 1.47$

- Condition for total internal reflection, $\boldsymbol{i > i_c}$
- From Snell's law, $n_1 \sin i = n_2 \sin r$
- When $\boldsymbol{i = i_c}$ ***then*** $\boldsymbol{r = 90°}$ Hence ,

$$n_1 \sin i_c = n_2 \sin 90° \quad (or) \quad n_1 \sin i_c = n_2 \quad (or) \quad \sin i_c = \frac{n_2}{n_1}$$

- Here,, $n_1 = n$ and $n_2 = 1$ So, $\sin i_c = \frac{1}{n}$ (or)

$$n = \frac{1}{\sin 45°} = \frac{1}{1/\sqrt{2}} = \sqrt{2} = 1.414$$

- Hence, $n_R < n$ So red colour will emerge out of the prism
- But, $n_G > n$ and $n_B > n$ So green and blue undergo total internal reflection

**4. An object is placed at a certain distance from a convex lens of focal length 20 cm. Find the object distance if the image obtained is magnified 4 times.**

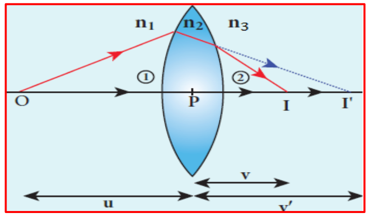
**Solution :** $f = 20\ cm = 20\ X\ 10^{-2}\ m$ ; $m = 4$

- If u be the object distance, then magnification,

$$m = \frac{h_2}{h_1} = \frac{f}{f+u} \qquad (or) \qquad f + u = \frac{f}{m}$$

$$(or) \qquad u = \frac{f}{m} - f = \frac{20}{4} - 20 = 5 - 20 = \boldsymbol{-15\ cm}$$

**5. Obtain the lens maker's formula for a lens of refractive index n₂ which is separating two media of refractive indices n₁ and n₃ on the left and right respectively.**




**Solution :**

- For the refracting surface ①, the light goes from $n_1$ to $n_2$, then

$$\frac{n_2}{v^1} - \frac{n_1}{u} = \frac{n_2 - n_1}{R_1} \quad ------ \quad (1)$$

- For the refracting surface ②, the light goes from $n_2$ to $n_3$, then

$$\frac{n_3}{v} - \frac{n_2}{v^1} = \frac{n_3 - n_2}{R_1} \quad ------ \quad (2)$$

- Adding equations (1) and (2)

$$\frac{n_2}{v^1} - \frac{n_1}{u} + \frac{n_3}{v} - \frac{n_2}{v^1} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_1}$$

$$\boldsymbol{\frac{n_3}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R_1} + \frac{(n_3 - n_2)}{R_1}}$$

**6. A thin converging lens of refractive index 1.5 has a power of + 5.0 *D*. When this lens is immersed in a liquid of refractive index n, it acts as a divergent lens of focal length 100 cm. What must be the value of *n*?**

**Solution :** $n_g = 1.5$ ; $P_g = +5.0\ D$ ; $f = -100\ cm = -1m$ ; $n_a = 1$

- Power of lens placed in water,

$$P_l = \frac{1}{f} = \frac{1}{(-1)} = = -1.0\ D$$

- When glass lens place in air

$$P_g = \left(\frac{n_g}{n_a} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ------ \quad (1)$$

- When glass lens immersed in liqiud

$$P_l = \left(\frac{n_g}{n} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right] \quad ------ \quad (2)$$

$\frac{(1)}{(2)} \Rightarrow$

$$\frac{P_g}{P_l} = \frac{\left(\frac{n_g}{n_a} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}{\left(\frac{n_g}{n} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]} = \frac{\left(\frac{n_g}{n_a} - 1\right)}{\left(\frac{n_g}{n} - 1\right)}$$

$$\frac{5.0}{(-1.0)} = \frac{\left(\frac{1.5}{1} - 1\right)}{\left(\frac{1.5}{n} - 1\right)}$$

$$-5 = \frac{(1.5 - 1)}{\left(\frac{1.5}{n} - 1\right)} = \frac{(0.5)}{\left(\frac{1.5}{n} - 1\right)}$$

$$\frac{1.5}{n} - 1 = \frac{(0.5)}{-5} = -0.1$$

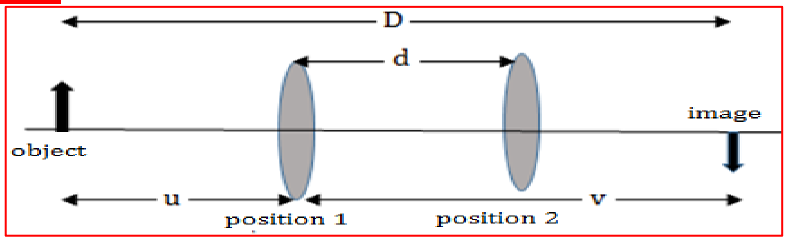
$$\frac{1.5}{n} = -0.1 + 1 = 0.9$$

$$\frac{1}{n} = \frac{0.9}{1.5} = \frac{9}{15} = \frac{3}{5}$$

$$\boldsymbol{n} = \frac{\mathbf{5}}{\mathbf{3}}$$

**7. If the distance *D* between an object and screen is greater than 4 times the focal length *f* of a convex lens, then there are two positions for which the lens forms an enlarged image and a diminished image respectively. This method is called conjugate foci method. If *d* is the distance between the two positions of the lens, obtain the equation for focal length of the convex lens.**

**Solution :**




- From figure,

$$D = u + v \quad ------ \quad (1)$$
$$d = v - u \quad ------ \quad (2)$$

- (1) + (2) $\quad D + d = u + v + v - u = 2v$

$$v = \frac{D + d}{2}$$

- (1) - (2) $\quad D - d = u + v - v + u = 2u$

$$u = \frac{D - d}{2}$$

- If 'f' is the focal length of convex lens,,

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{(-u)} = \frac{1}{v} + \frac{1}{u} = \frac{u + v}{u\,v}$$

$$f = \frac{u\,v}{u + v}$$

- *Put the value of u and v,*

$$f = \frac{\left(\frac{D - d}{2}\right)\left(\frac{D + d}{2}\right)}{\frac{D + d}{2} + \frac{D - d}{2}} = \frac{\left[\frac{(D + d)(D - d)}{4}\right]}{\left[\frac{D + d + D - d}{2}\right]}$$

$$\boldsymbol{f} = \frac{(\boldsymbol{D} + \boldsymbol{d})(\boldsymbol{D} - \boldsymbol{d})}{\mathbf{4}\,\boldsymbol{D}}$$

$$\boldsymbol{f} = \frac{\boldsymbol{D}^2 - \boldsymbol{d}^2}{\mathbf{4}\,\boldsymbol{D}}$$

**8. Prove that a convex mirror can only form a virtual, erect and diminished image.**

**Solution :**

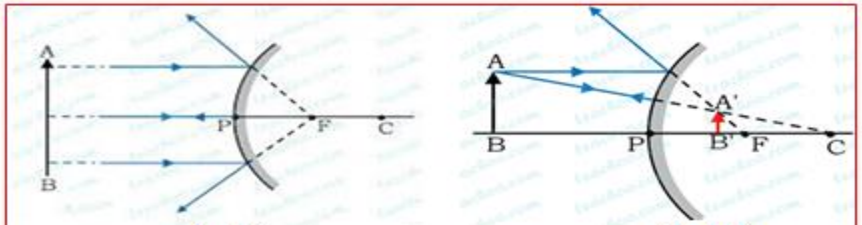



fig (i) fig (ii)

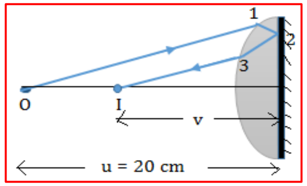
**Figure (i):**

- Position of object - At infinity
- Position of image- At F, right side of convex mirror
- Size of the image - Point image
- Nature of the image = Erect, diminished and virtual image

**Figure (ii):**

- Position of object - Between pole (P) and infinity
- Position of image- Between Pole (P) and Focus (F) on right side of convex mirror
- Size of the image - very small
- Nature of the image = Erect, diminished and virtual image

**9. A point object is placed at 20 cm from a thin plano-convex lens of focal length 15 cm whose plane surface is silvered. Locate the position and nature of the final image.**




**Solution :** $\because f_{lens} = 15\ cm\ ; \quad u = 20\ cm$

- The light from'O'undergoes two refractions (1,3) and one reflection (2) and forms final image at 'I'
- Hence the equivalent power of this combination,

$$P_{equivalent} = 2P_{lens} + P_{mirror}$$

$$-\frac{1}{f_{equivalent}} = \frac{2}{f_{convex}} + \frac{1}{f_{mirror}}$$

$$-\frac{1}{f_{equivalent}} = \frac{2}{15} + \frac{1}{\infty} = -\frac{2}{15} \qquad \left[\because \frac{1}{\infty} = 0\right]$$

$$\therefore \quad \boldsymbol{f_{equivalent} = -\frac{15}{2}\ cm}$$

- From mirror equation; $\frac{1}{v} + \frac{1}{u} = \frac{1}{f_{equivalent}}$

$$\therefore \frac{1}{v} = \frac{1}{f_{equivalent}} - \frac{1}{u} = -\frac{2}{15} - \frac{1}{(-20)} = -\frac{2}{15} + \frac{1}{20} = \frac{-8+3}{60} = \frac{-5}{60} = -\frac{1}{12}$$

$$\boldsymbol{v = -12\ cm}$$

- Hence final image will form at 12 cm left side of the system.

**10. Find the ratio of the intensities of lights with wavelengths 500 nm and 300 nm which undergo Rayleigh scattering.**

**Solution :** $\lambda_1 = 500\ nm = 500\ X10^{-9}m \quad ; \quad \lambda_2 = 300\ nm = 300\ X10^{-9}m$

- From Rayleigh's scattering law, the intensity of scattered light ; $I \propto \frac{1}{\lambda^4}$
- Hence, $I_1 \propto \frac{1}{\lambda_1^4}$ *and* $I_2 \propto \frac{1}{\lambda_2^4}$
- From this,

$$\frac{I_1}{I_2} = \frac{\lambda_2^4}{\lambda_1^4} = \left(\frac{300\ X10^{-9}}{500\ X10^{-9}}\right)^4 = \left(\frac{3}{5}\right)^4 = \frac{81}{625}$$

$$(or) \qquad \boldsymbol{I_1 : I_2 = 81 : 625}$$

**11. Refractive index of material of the prism is 1.541. Find the critical angle?**

**Solution :** $\boldsymbol{n = 1.541}$

- Let $i_C$ be the critical angle, then

$$\sin i_C = \frac{1}{n} = \frac{1}{1.541} = 0.6489$$

$$\therefore \quad \boldsymbol{i_C} = \sin^{-1}(0.6489) = \boldsymbol{42°27'}$$

| No | Log |
|---|---|
| 1<br>1.541 | 0.0000<br>0.1878 |
| (-)<br>ALog | 1.8122<br>$6.489\ X\ 10^{-1}$ |