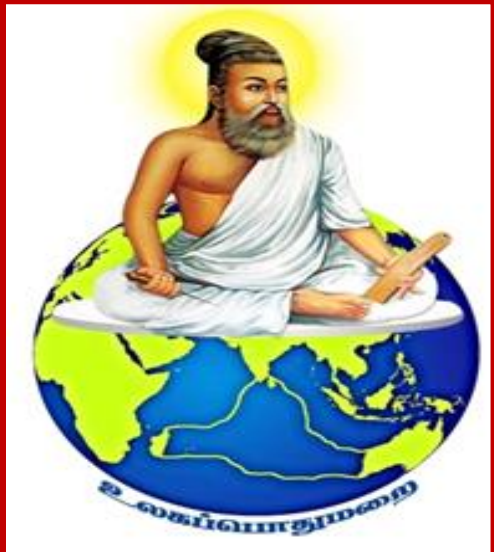
HIGHER SECONDARY
SECOND YEAR

PHYSICS



UNIT -4

ELECTROMAGNETIC INDUCTION
AND
ALTERNATING CURRENT

PROBLEMS AND SOLUTIONS

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EXAMPLE PROBLEMS

1. A circular antenna of area 3 m² is installed at a place in Madurai. The plane of the area of antenna is inclined at 47° with the direction of Earth's magnetic field. If the magnitude of Earth's field at that place is 4.1 × 10⁻⁵ T find the magnetic flux linked with the antenna.

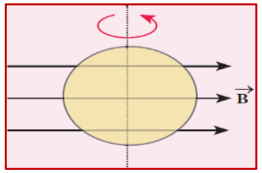
Solution : $A = 3\,m^2$; $\theta = 90° - 47° = 43°$; $B = 4.1 \text{ X } 10^{-5}$ T

♠ Magnetic flux,

$$\Phi_B = B\,A\cos\theta$$
$$\Phi_B = 4.1 \text{ X } 10^{-5} \text{ X } 3 \text{ X } \cos 43°$$
$$\Phi_B = 4.1 \text{ X } 10^{-5} \text{ X } 3 \text{ X } 0.7314$$
$$\Phi_B = 8.997 \text{ X } 10^{-5} =$$
$$\boldsymbol{\Phi_B = 89.97 \text{ X } 10^{-6}\,Wb = 89.97\,\mu Wb}$$

No	Log
4.1	0.6128
3	0.4771
0.7314	$\bar{1}$.8642
+	0.9541
Alog	$8.997 \text{ X } 10^0$

2. A circular loop of area 5 ×10⁻² m² rotates in a uniform magnetic field of 0.2 T. If the loop rotates about its diameter which is perpendicular to the magnetic field as shown in figure. Find the magnetic flux linked with the loop when its plane is (a) normal to the field (b) inclined 60° to the field and (c) parallel to the field.



Solution : $A = 5\,X\,10^{-2}\,m^2$; $B = 0.2\,T$

(a) When circular loop normal to the magnetic field, then $\boldsymbol{\theta = 0°}$. The magnetic flux

$$\Phi_B = B\,A\cos\theta$$
$$\Phi_B = 0.2\,X\,5\,X\,10^{-2} \text{ X } \cos 0°$$
$$\Phi_B = 1\,X\,10^{-2} \text{ X } 1 = \mathbf{1\,X\,10^{-2}\,Wb}$$

(b) When circular loop inclined **60°** to the magnetic field, then $\boldsymbol{\theta = 90° - 60° = 30°}$ The magnetic flux

$$\Phi_B = B\,A\cos\theta$$
$$\Phi_B = 0.2\,X\,5\,X\,10^{-2} \text{ X } \cos 30°$$
$$\Phi_B = 1\,X\,10^{-2} \text{ X } \frac{\sqrt{3}}{2} = \frac{1.732}{2}\,X\,10^{-2}$$
$$\boldsymbol{\Phi_B = 0.866\,X10^{-2} = 8.66\,X\,10^{-3}\,\text{Wb}}$$

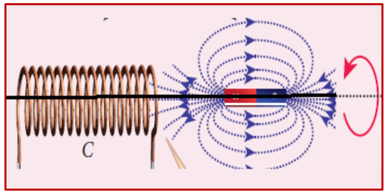
(c) When circular loop parallel to the magnetic field, then $\boldsymbol{\theta = 90°}$. The magnetic flux

$$\Phi_B = B\,A\cos\theta$$
$$\Phi_B = 0.2\,X\,5\,X\,10^{-2} \text{ X } \cos 90°$$
$$\Phi_B = 1\,X\,10^{-2} \text{ X } 0 = \mathbf{0}$$

3. A cylindrical bar magnet is kept along the axis of a circular solenoid. If the magnet is rotated about its axis, find out whether an electric current is induced in the coil.

Solution :

♠ The magnetic field of a cylindrical magnet is symmetrical about its axis. As the magnet is rotated along the axis of the solenoid, there is no induced current in the solenoid because the flux linked with the solenoid does not change due to the rotation of the magnet



4. A closed coil of 40 turns and of area 200 cm², is rotated in a magnetic field of flux density 2 Wb m⁻². It rotates from a position where its plane makes an angle of 30° with the field to a position perpendicular to the field in a time 0.2 s. Find the magnitude of the emf induced in the coil due to its rotation.

Solution : $A = 200\,cm^2 = 200\,X\,10^{-4}\,m^2$; $B = 2\,T$; $N = 40$; $t = 0.2\,s$

♠ Initially, $\theta = 90° - 30° = 60$; Hence initial magnetic flux

$$\Phi_{B_i} = B\,A\cos\theta$$
$$\Phi_{B_i} = 2\,X\,200\,X\,10^{-4} \text{ X } \cos 60°$$
$$\Phi_{B_i} = 400\,X\,10^{-4} \text{ X } \frac{1}{2} = 200\,X\,10^{-4}$$
$$\Phi_{B_i} = 2\,X\,10^{-2}\,Wb$$

♠ Finally, $\theta = 90° - 90° = 0°$; Hence final magnetic flux

$$\Phi_{B_f} = B\,A\cos\theta$$
$$\Phi_{B_f} = 2\,X\,200\,X\,10^{-4} \text{ X } \cos 0°$$
$$\Phi_{B_f} = 400\,X\,10^{-4} \text{ X } 1 = 400\,X\,10^{-4}$$
$$\Phi_{B_f} = 4\,X\,10^{-2}\,Wb$$

♠ Since the magnetic flux changes, an emf is induced which is given by

$$\epsilon = N\,\frac{d\Phi_B}{dt} = N\,\frac{\Phi_{B_f} - \Phi_{B_i}}{t}$$
$$\epsilon = 40\,X\,\frac{4\,X\,10^{-2} - 2\,X\,10^{-2}}{0.2} = \frac{40\,X\,2\,X\,10^{-2}}{0.2}$$
$$\epsilon = 400\,X\,10^{-2} = \mathbf{4\,V}$$

5. A straight conducting wire is dropped horizontally from a certain height with its length along east – west direction. Will an emf be induced in it? Justify your answer.

Solution :

♠ Yes! An emf will be induced in the wire because it moves perpendicular to the horizontal component of Earth's magnetic field and hence it cuts the magnetic lines of Earth's magnetic filed.

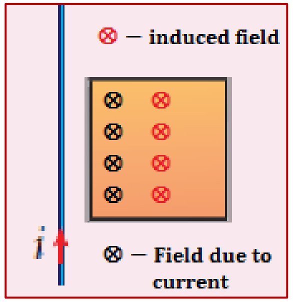
6. If the current *i* flowing in the straight conducting wire as shown in the figure decreases, find out the direction of induced current in the metallic square loop placed near it.

Solution :

♠ From right hand rule, the magnetic field by the straight wire is directed into the plane of the square loop perpendicularly and its magnetic flux is decreasing.

♠ The decrease in flux is opposed by the current induced in the loop by producing a magnetic field in the same direction as the magnetic field of the wire.

♠ Again from right hand rule, for this inward magnetic field, the direction of the induced current in the loop is clockwise.



7. The magnetic flux passes perpendicular to the plane of the circuit and is directed into the paper. If the magnetic flux varies with respect to time as per the following relation: $\Phi_B = (2t^3 + 3t^2 + 8t + 5)$ mWb , what is the magnitude of the induced emf in the loop when t = 3 s? Find out the direction of current through the circuit.

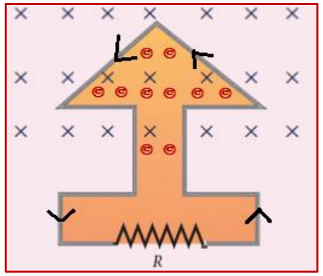
Solution : $\Phi_B = (2\,t^3 + 3\,t^2 + 8\,t + 5)\ X\ 10^{-3}\ Wb$; $N = 1$; $t = 3\,s$; $\in\ = ?$; $i = ?$

- From laws of electromagnetic induction,

$$\in\ = N\frac{d\Phi_B}{dt} = \frac{d}{dt}(2\,t^3 + 3\,t^2 + 8\,t + 5)\ X\ 10^{-3}$$
$$\in\ = (2\ X\ 3\,t^2 + 3\ X\ 2\,t + 8 + 0)X\ 10^{-3}$$
$$\in\ = (6\,t^2 + 6\,t + 8)\ X\ 10^{-3}$$

- At **t = 3 s**, the magnitude of induced emf

$$\in\ = [6\,(3)^2 + 6\,(3) + 8]X\ 10^{-3}$$
$$\in\ = [54 + 18 + 8]X\ 10^{-3}$$
$$\in\ = \mathbf{80\ X\ 10^{-3}\ V = 80\ m\ V}$$



- As time passes, the magnetic flux linked with the loop increases.
- According to Lenz's law, the direction of the induced current should be in a way so as to oppose the flux increase.
- So, the induced current flows in such a way to produce a magnetic field opposite to the given field. This magnetic field is perpendicularly outwards.
- Therefore, the induced current flows in anti-clockwise direction.

8. A conducting rod of length 0.5 m falls freely from the top of a building of height 7.2 m at a place in Chennai where the horizontal component of Earth's magnetic field is 4.04 × 10⁻⁵ T. If the length of the rod is perpendicular to Earth's horizontal magnetic field, find the emf induced across the conductor when the rod is about to touch the ground. (Assume that the rod falls down with constant acceleration of 10 m s⁻²)

Solution : $B_H = 4.04\ X\ 10^{-5}\ T$; $h = 7.2\ m$; $l = 0.5\ m$

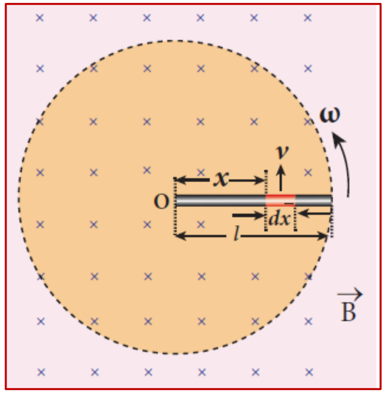
- From the equation of motion, the final velocity of the rod is

$$v^2 = u^2 + 2\,g\,h \qquad [\because u = 0]$$
$$v^2 = 0 + (2\ X\ 10\ X\ 7.2)$$
$$v^2 = 144$$
$$\boldsymbol{v = 12\ m\ s^{-1}}$$

- The magnitude of the induced emf when the rod is about to touch the ground is

$$\in\ = B_H\,l\,v$$
$$\in\ = 4.04\ X\ 10^{-5}\ X\ 0.5\ X\ 12$$
$$\in\ = 24.24\ X\ 10^{-5}\ V$$
$$\in\ = \mathbf{242.4\ X\ 10^{-6}\ V = 242.4\ \mu\ V}$$

9. A copper rod of length l rotates about one of its ends with an angular velocity ω in a magnetic field B as shown in the figure. The plane of rotation is perpendicular to the field. Find the emf induced between the two ends of the rod.



Solution :

- Consider a small element of length dx at a distance x from the centre O of the circle described by the rod.
- As this element moves perpendicular to the field with a linear velocity $v = x\,\omega$, the emf developed in the element dx is

$$d\in\ = B\ dx\ v = B\ dx\ (x\ \omega)$$
$$d\in\ = B\ \omega\ x\ dx$$

- This rod is made up of many such elements, moving perpendicular to the field. The emf developed across two ends is

$$\in\ = B\,\omega\int_0^l x\,dx = B\,\omega\left[\frac{x^2}{2}\right]_0^l$$
$$\in\ = \frac{\mathbf{1}}{\mathbf{2}}\,\boldsymbol{B\,\omega\,l^2}$$

10. A solenoid of 500 turns is wound on an iron core of relative permeability 800. The length and radius of the solenoid are 40 cm and 3 cm respectively. Calculate the average emf induced in the solenoid if the current in it changes from 0 to 3 A in 0.4 second.

Solution : : $\mu_r = 800$; $N = 500$; $l = 40\ cm = 40\ X\ 10^{-2} m$; $r = 3\ cm = 3\ X\ 10^{-2} m$; $di = 3 - 0 = 3\ A$; $dt = 0.4\ s$; $\in\ = ?$

- Self inductance,

$$L = \frac{\mu_0\,\mu_r\,N^2\,A}{l} = \frac{\mu_0\,\mu_r\,N^2\,\pi\,r^2}{l}$$
$$L = \frac{4\,\pi\ X\ 10^{-7}\ X\ 800\ X\ (500)^2\ X\ 3.14\ X\ (3\ X\ 10^{-2})^2}{40\ X\ 10^{-2}}$$
$$L = \frac{4\ X\ 3.14\ X\ 10^{-7}\ X\ 800\ X\ 250000\ X\ 3.14\ X\ 9\ X\ 10^{-4}}{40\ X\ 10^{-2}}$$
$$L = 4\ X\ 3.14\ X\ 20\ X\ 250000\ X\ 3.14\ X\ 9\ X\ 10^{-9}$$
$$L = 4\ X\ 3.14\ X\ 2\ X\ 25\ X\ 3.14\ X\ 9\ X\ 10^{-4}$$
$$L = 3.14\ X\ 3.14\ X\ 1800\ X\ 10^{-4}$$
$$L = 1.775\ X\ 10^4\ X\ 10^{-4}$$
$$\boldsymbol{L = 1.775\ H}$$

No	Log
3.14	0.4969
3.14	0.4969
1800	3.2553
(+)	4.2491
ALog	$1.775\ X\ 10^4$

- Hence induced emf,

$$\in\ = -L\frac{di}{dt} = -1.775\ X\ \frac{3}{0.4} = -1.775\ X\ \frac{30}{4} = -1.775\ X\ 7.5$$
$$\in\ = \mathbf{-13.3125\ V}$$

11. The self-inductance of an air-core solenoid is 4.8 mH. If its core is replaced by iron core, then its self-inductance becomes 1.8 H. Find out the relative permeability of iron.

Solution : $L_{air} = 4.8\ mH = 4.8\ X\ 10^{-3}\ H$; $L_{iron} = 1.8\ H$; $\mu_r = ?$

- Self inductance of air core solenoid ;

$$L_{aie} = \frac{\mu_0\ N^2\ A}{l}$$

- Self inductance of iron core solenoid ;

$$L_{iron} = \frac{\mu_0\ \mu_r\ N^2\ A}{l}$$

- Hence,

$$\frac{L_{air}}{L_{iron}} = \frac{\left(\frac{\mu_0\ N^2\ A}{l}\right)}{\left(\frac{\mu_0\ \mu_r\ N^2\ A}{l}\right)} = \frac{1}{\mu_r}$$

$$\therefore \quad \mu_r = \frac{L_{iron}}{L_{air}} = \frac{1.8}{4.8\ X\ 10^{-3}} = \frac{3\ X\ 10^3}{8} = \frac{3000}{8}$$

$$\boldsymbol{\mu_r = 375\ (no\ unit)}$$

12. The current flowing in the first coil changes from 2 A to 10 A in 0.4 s. Find the mutual inductance between two coils if an emf of 60 mV is induced in the second coil. Also determine the magnitude of induced emf in the second coil if the current in the first coil is changed from 4 A to 16 A in 0.03 s. Consider only the magnitude of induced emf.

Solution :

(i) $di_1 = 10 - 2 = 8\ A$; $dt = 0.4\ s$; $\epsilon_2 = 60\ mV = 60\ X\ 10^{-3}\ V$; $M = ?$

- Magnitude of mutual induced emf is ;

$$\epsilon_2 = M_{21}\frac{di_1}{dt}$$

- Hence mutual inductance between the coils,

$$M_{21} = \frac{\epsilon_2}{\left(\frac{di_1}{dt}\right)}$$

$$M_{21} = \frac{60\ X\ 10^{-3}}{\left(\frac{8}{0.4}\right)} = \frac{60\ X\ 10^{-3}\ X\ 0.4}{8} = 60\ X\ 10^{-3}\ X\ 0.05$$

$$\boldsymbol{M_{21} = 3\ X\ 10^{-3}\ H = 3\ m}\ H$$

(ii) $di_1 = 16 - 4 = 12\ A$; $dt = 0.03\ s$; $\epsilon_2 = ?$

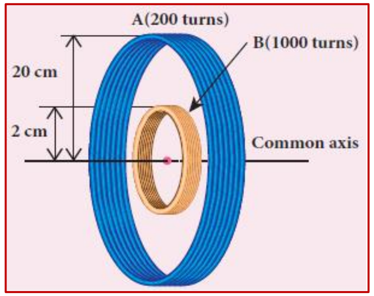
- Magnitude of Induced emf in the second coil due to the rate of change of current in the first coil is

$$\epsilon_2 = M_{21}\frac{di_1}{dt}$$

$$\epsilon_2 = 3\ X\ 10^{-3}\ X\ \frac{12}{0.03} = 100\ X\ 10^{-3}\ X\ 12$$

$$\epsilon_2 = \mathbf{1.2\ V}$$

13. Consider two coplanar, co-axial circular coils *A* and *B* as shown in figure. The radius of coil *A* is 20 cm while that of coil *B* is 2 cm. The number of turns in coils *A* and *B* are 200 and 1000 respectively. Calculate the mutual inductance between the coils. If the current in coil *A* changes from 2 A to 6 A in 0.04 s, determine the induced emf in coil *B* and the rate of change of flux through the coil *B* at that instant.



Solution : $N_A = 200$; $N_B = 1000$; $r_A = 20cm = 20\ X\ 10^{-2}\ m$; $r_B = 2\ cm = 2\ X10^{-2}m$

$di_A = 6 - 2 = 4\ A$; $dt = 0.04\ s$; $M_{BA} = ?$; $\epsilon_B = ?$

- Mutual inductance between the coils,

$$M_{BA} = \frac{N_B\ \Phi_B}{i_A} = \frac{N_B\ B_A\ A_B}{i_A} = \frac{N_B\ \left(\frac{\mu_0\ N_A\ i_A}{2\ r_A}\right)\ \pi\ r_B^2}{i_A} = \frac{N_B\ \mu_0\ N_A\ i_A\ \pi\ r_B^2}{2\ r_A\ i_A}$$

$$M_{BA} = \frac{\mu_0\ N_A\ N_B\ \pi\ r_B^2}{2\ r_A}$$

$$M_{BA} = \frac{4\ \pi\ X\ 10^{-7}\ X\ 200\ X\ 1000\ X\ \pi\ X\ (2\ X10^{-2})^2}{2\ X\ 20\ X\ 10^{-2}}$$

$$M_{BA} = 8\ X\ 3.14\ X\ 3.14\ X\ 10^{-5}$$

$$M_{BA} = 7.887\ X\ 10^1\ X\ 10^{-5} = \mathbf{7.887\ X\ 10^{-4}\ H}$$

No	Log
3.14 3.14 8	0. 4969 0. 4969 0. 9031
(+) ALog	1. 8969 7. 887 X 10^1

- Magnitude of the induced emf in the coil B,

$$\epsilon_B = M_{BA}\frac{di_A}{dt} = 7.887\ X\ 10^{-4}\ X\ \frac{4}{0.04} = 7.887\ X\ 10^{-4} X\ 100$$

$$\epsilon_B = 7.887\ X\ 10^{-2}\ V = 78.87\ X\ 10^{-3}\ V = \mathbf{78.87\ mV}$$

- The rate of change of magnetic flux of coil *B* is

$$\frac{\boldsymbol{d}}{\boldsymbol{dt}}(N_B\Phi_B) = \epsilon_B = 78.87\ X\ 10^{-3}\ V = \mathbf{78.87\ m\ Wb\ s^{-1}}$$

14. A circular metal of area 0.03 m² rotates in a uniform magnetic field of 0.4 T. The axis of rotation passes through the centre and perpendicular to its plane and is also parallel to the field. If the disc completes 20 revolutions in one second and the resistance of the disc is 4 Ω, calculate the induced emf between the axis and the rim and induced current flowing in the disc.

Solution : : $B = 0.4\ T$; $A = 0.3\ m^2$; $f = 20\ rps$; $R = 4\ \Omega$; $\epsilon = ?$; $i = ?$

- Area swept out by the disc in unit time = Area of the disc × frequency

$$\frac{dA}{dt} = 0.3\ X\ 20 = 0.6\ m^2$$

- Hence induced emf,

$$\epsilon = \frac{d\Phi_B}{dt} = \frac{d\ (B\ A)}{dt} = B\frac{dA}{dt} = 0.4\ X\ 0.6 = \mathbf{0.24\ V}$$

- Thus induced current,

$$\boldsymbol{i} = \frac{\epsilon}{R} = \frac{0.24}{4} = \mathbf{0.06\ A}$$

15. A rectangular coil of area 70 cm² having 600 turns rotates about an axis perpendicular to a magnetic field of 0.4 Wb m⁻². If the coil completes 500 revolutions in a minute, calculate the instantaneous emf when the plane of the coil is (a) perpendicular to the field (b) parallel to the field and (c) inclined at 60° with the field.

Solution : $N = 600\,;A = 70\,cm^2 = 70\,X\,10^{-4}\,m^2\,;B = 0.4\,T$;

$$f = 500\,rpm = \frac{500}{60} = \frac{50}{6} = 8.333\,rps$$

(a) When perpendicular to the field, $\theta = \omega t = 0°$

$$\epsilon = \epsilon_m \sin\omega t = N\,B\,A\,\omega \sin\omega t = N\,B\,A\,2\pi f \sin\omega t$$

$$\epsilon = 600\,X\,0.4\,X\,70\,X\,10^{-4}\,X\,2\,\pi\,X\,\frac{50}{6}\,X\,\sin 0° = \mathbf{0}$$

(b) When parallel to the field, $\theta = \omega t = 90°$

$$\epsilon = \epsilon_m \sin\omega t = N\,B\,A\,\omega \sin\omega t = N\,B\,A\,2\pi f \sin\omega t$$

$$\epsilon = 600\,X\,0.4\,X\,70\,X\,10^{-4}\,X\,2\,\pi\,X\,\frac{50}{6}\,X\,\sin 90°$$

$$\epsilon = 100\,X\,0.4\,X\,70\,X\,2\,X\,\frac{22}{7}X\,50\,X\,1\,X\,10^{-4}$$

$$\epsilon = 88\,X\,10^4\,X\,10^{-4} = \mathbf{88\ V}$$

(c) When inclined at 60° with the field, $\theta = \omega t = 90° - 60° = 30°$

$$\epsilon = \epsilon_m \sin\omega t = N\,B\,A\,\omega \sin\omega t = N\,B\,A\,2\pi f \sin\omega t$$

$$\epsilon = 600\,X\,0.4\,X\,70\,X\,10^{-4}\,X\,2\,\pi\,X\,\frac{50}{6}\,X\,\sin 30°$$

$$\epsilon = 100\,X\,0.4\,X\,70\,X\,2\,X\,\frac{22}{7}X\,50\,X\,1\,X\,10^{-4}\,X\,\frac{1}{2}$$

$$\epsilon = 44\,X\,10^4\,X\,10^{-4} = \mathbf{44\ V}$$

16. An ideal transformer has 460 and 40,000 turns in the primary and secondary coils respectively. Find the voltage developed per turn of the secondary if the transformer is connected to a 230 V AC mains. The secondary is given to a load of resistance 10⁴ Ω. Calculate the power delivered to the load.

Solution : $N_P = 460\,;\ N_S = 40000\,;\ V_P = 230\,V\ ;\ R_S = 10^4\,\Omega\ ;\ \frac{V_S}{N_S} = ?\ ;\ P = ?$

- From the transformer equation, voltage per turn of the secondary is ;

$$\frac{V_S}{N_S} = \frac{V_P}{N_P} = \frac{230}{460} = \frac{1}{2}$$

$$\therefore \quad \frac{V_S}{N_S} = \mathbf{0.5\,V/turn}$$

- Total secondary voltage ; $V_S = N_S\,X\,0.5 = 40000\,X\,0.5 = \mathbf{20000\,V}$
- Power delivered to the load ,

$$P_S = V_S\,I_S = V_S\,\frac{V_S}{R_S}$$

$$P_S = \frac{20000\,X\,20000}{10^4} = 40000\,W$$

$$\mathbf{P_S = 40\,kW}$$

17. An inverter is common electrical device which we use in our homes. When there is no power in our house, inverter gives AC power to run a few electronic appliances like fan or light. An inverter has inbuilt step-up transformer which converts 12 V AC to 240 V AC. The primary coil has 100 turns and the inverter delivers 50 mA to the external circuit. Find the number of turns in the secondary and the primary current.

Solution : $V_P = 12\,V\ ;\ V_S = 240\,V\ ;\ N_P = 100\,V\ ;\ I_S = 50\,mA = 50\,X\,10^{-3}\,A$

- By transformer equation;.

$$\frac{V_S}{V_S} = \frac{N_S}{N_P} = \frac{I_P}{I_S} = K$$

- Hence the transformation ratio ;

$$K = \frac{V_S}{V_S} = \frac{240}{12} = 20$$

- Number of turns in secondary coil ; $N_S = N_P\,K = 100\,X\,20 = \mathbf{2000}$
- Primary current ; $I_P = I_S\,K = 50\,X\,10^{-3}\,X\,20 = 1000\,X\,10^{-3} = \mathbf{1\,A}$

18. Write down the equation for a sinusoidal voltage of 50 Hz and its peak value is 20 V. Draw the corresponding voltage versus time graph.

Solution : $f = 50\,Hz\ ;\ V_m = 20\,V\ ;\ V = ?\ ;\ T = ?$

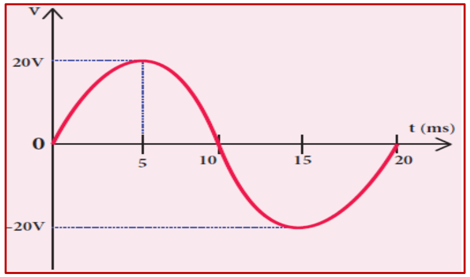
- Voltage at any instant,

$$V = V_m \sin\omega t = V_m \sin 2\pi f t$$
$$V = 20\sin(2\,X\,3.14\,X\,50\,X\,t)$$
$$\mathbf{V = 20\,\sin 314\,t}$$

- Time for one cycle ,

$$T = \frac{1}{f} = \frac{1}{50} = 0.02\,s$$
$$\mathbf{T = 20\,X\,10^{-3}\,s = 20\,ms}$$

- Corresponding Wave form is shown



19. The equation for an alternating current is given by *i* = 77 *sin* 314*t*. Find the peak current, frequency, time period and instantaneous value of current at *t* = 2 ms.

Solution : : $i = 77\sin 314\,t\ ;\ t = 2\,ms = 2\,X\,10^{-3}\,s$

- General equation for alternating current ; $i = I_m \sin\omega t = I_m \sin 2\pi f t$
- Comparing this equation with given equation, we get

(a) Peak current ; $\mathbf{I_m = 77\,A}$

(b) Frequency ; $f = \frac{\omega}{2\pi} = \frac{314}{2\pi} = \frac{314}{2\,X\,3.14} = \frac{100}{2} = \mathbf{50\,Hz}$

(c) Time period ; $T = \frac{1}{f} = \frac{1}{50} = \mathbf{0.02\,s}$

(d) At $t = 2\,ms$, instantaneous current

$$i = 77\sin(314\,X\,2\,X\,10^{-3}\,rad)$$
$$i = 77\sin\left(314\,X\,2\,X\,10^{-3}\,X\,\frac{180°}{3.14}\right)$$
$$i = 77\,X\,\sin 36°$$
$$\mathbf{i = 77\,X\,0.5877 = 45.26\,A}$$

No	Log
77	1.8865
0.5878	$\bar{1}.7692$
(+)	1.6557
ALog	$4.526\,X\,10^1$

20. A 400 mH coil of negligible resistance is connected to an AC circuit in which an effective current of 6 mA is flowing. Find out the voltage across the coil if the frequency is 1000 Hz.

Solution : $I_{eff} = 6\,mA = 6 X 10^{-3}\,A$; $L = 400\,mH = 400 X 10^{-3}\,H$; $f = 1000\,Hz$

- Voltage across the coil of inductance L

$$V_L = I X_L = I\,\omega\,L = I\,(2\,\pi\,f)\,L$$
$$V_L = 6 X 10^{-3} X 2 X 3.14 X 1000 X 400 X 10^{-3}$$
$$V_L = 150.72 X 10^{-1}$$
$$\boldsymbol{V_L = 15.072\ V}$$

21. A capacitor of capacitance $\frac{10^2}{\pi}$ μF is connected across a 220 V, 50 Hz A.C. mains. Calculate the capacitive reactance, RMS value of current and write down the equations of voltage and current.

Solution : $V_{RMS} = 220\,V$; $f = 50\,Hz$; $C = \frac{10^2}{\pi}\,\mu F = \frac{10^2}{\pi} X 10^{-6}\,F$; $X_C = ?$; $I_{RMS} = ?$

- Capacitive reactance ; $X_C = \frac{1}{\omega\,C} = \frac{1}{2\,\pi\,f\,C}$

$$X_C = \frac{1}{2 X \pi X 50 X \left(\frac{10^2}{\pi}\right) X 10^{-6}} = \frac{1}{10^{-2}}$$
$$\boldsymbol{X_C = 10^2\ \Omega = 100\ \Omega}$$

- RMS value of alternating current ; $\boldsymbol{I_{RMS}} = \frac{V_{RMS}}{X_C} = \frac{220}{100} = \boldsymbol{2.2\ \Omega}$
- Equation for alternating voltage ;

$$V = V_m \sin\omega t$$
$$V = V_{RMS}\sqrt{2}\,\sin 2\,\pi\,f\,t$$
$$V = 220 X 1.414\ \sin 2 X 3.14 X 50 X t$$
$$\boldsymbol{V = 311\ \sin 314\,t}$$

- Equation for alternating current ;

$$i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$
$$i = I_{RMS}\sqrt{2}\,\sin\left(2\,\pi\,f\,t + \frac{\pi}{2}\right)$$
$$i = 2.2 X 1.414 \sin\left(2 X 3.14 X 50 X t + \frac{\pi}{2}\right)$$
$$\boldsymbol{i = 3.11\ \sin\left(314\,t + \frac{\pi}{2}\right)}$$

22. Find the impedance of a series *RLC* circuit if the inductive reactance, capacitive reactance and resistance are 184 Ω, 144 Ω and 30 Ω respectively. Also calculate the phase angle between voltage and current.

Solution : $X_L = 184\,\Omega$; $X_C = 144\,\Omega$; R = 30 Ω ; Z = ? ; ϕ = ?

- Impedance ; $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$Z = \sqrt{30^2 + (184 - 144)^2} = \sqrt{30^2 + 40^2}$$
$$Z = \sqrt{900 + 1600} = \sqrt{2500}$$
$$\boldsymbol{Z = 50\ \Omega}$$

- Phase angle between voltage and current ;

$$\tan\phi = \frac{X_L - X_C}{R}$$
$$\phi = \tan^{-1}\left[\frac{X_L - X_C}{R}\right] = \tan^{-1}\left[\frac{184 - 144}{30}\right]$$
$$\phi = \tan^{-1}\left[\frac{40}{30}\right] = \tan^{-1}\left[\frac{4}{3}\right]$$
$$\boldsymbol{\phi} = \tan^{-1}[1.333] = \boldsymbol{53.12°}$$

- Since the phase angle is positive, ***voltage leads current by 53.12°*** for this inductive circuit.

23. A 500 μH inductor, $\frac{80}{\pi^2}$ pF capacitor and a 628 Ω resistor are connected to form a series *RLC* circuit. Calculate the resonant frequency and Q-factor of this circuit at resonance.

Solution : $L = 500\,\mu H = 500 X 10^{-6}\,H$; $C = \frac{80}{\pi^2}\,pF = \frac{80}{\pi^2} X 10^{-12}\,F$; R = 628 Ω

- Resonance frequency,

$$f_r = \frac{1}{2\,\pi\sqrt{L\,C}} = \frac{1}{2\,\pi\sqrt{500 X 10^{-6} X \frac{80}{\pi^2} X 10^{-12}}} = \frac{1}{2\,\pi X \frac{10^{-9}}{\pi}\sqrt{500 X 80}}$$
$$f_r = \frac{10^9}{2\sqrt{500 X 80}} = \frac{10^9}{2\sqrt{40000}} = \frac{10^9}{2 X 200} = \frac{10^9}{400} = \frac{10^9}{4 X 10^2} = \frac{1}{4} X 10^7$$
$$f_r = 0.25 X 10^7 = 2500 X 10^3\,Hz = \boldsymbol{2500\ k\,Hz}$$

- Q –factor,

$$Q = \frac{V_L}{V_R} = \frac{I\,X_L}{I\,R} = \frac{X_L}{R} = \frac{\omega_r\,L}{R} = \frac{2\,\pi\,f_r\,L}{R}$$
$$Q = \frac{2 X 3.14 X 2500 X 10^3 X 500 X 10^{-6}}{628}$$
$$Q = \frac{2 X 2500 X 10^3 X 500 X 10^{-6}}{200} = \frac{25000 X 10^{-3}}{2}$$
$$\boldsymbol{Q = 12500 X 10^{-3} = 12.5}$$

24. Find the instantaneous value of alternating voltage $v = 10\sin(3\pi X 10^4\,t)$ volt at (a) 0 s (b) 50 μs (c) 75 μs.

Solution :

- Voltage at any instant ; $v = V_m \sin\omega\,t$
- Given voltage equation ; $v = 10\sin(3\pi X 10^4\,t)$

(a) At $t = 0\,s$; $v = 10\sin 0° = \mathbf{0}$

(b) At $t = 50\,\mu s$,

$$v = 10\sin(3\pi X 10^4 X 50 X 10^{-6})$$
$$v = 10\sin(150\,\pi X 10^{-2}\,rad)$$
$$v = 10\sin\left(150\,\pi X 10^{-2} X \frac{180°}{\pi}\right)$$
$$v = 10\sin(1.5 X 180°) = 10\sin(270°)$$
$$v = 10 X (-1) = \boldsymbol{-10\ V}$$

(c) At $t = 75\ \mu s$; $v = 10 \sin(3\pi\ X\ 10^4\ X\ 75\ X\ 10^{-6})$

$$v = 10 \sin(225\ \pi\ X\ 10^{-2}\ rad) = 10 \sin\left(225\ \pi\ X\ 10^{-2} X\ \frac{180°}{\pi}\right)$$
$$v = 10 \sin(2.25\ X\ 180°) = 10 \sin(405°)$$
$$v = 10 \sin(360 + 45°) = 10 \sin(45°)$$
$$v = 10\ X\ (0.7071) = \mathbf{7.071\ V}$$

25. The current in an inductive circuit is given by 0.3 sin (200*t* – 40°) A. Write the equation for the voltage across it if the inductance is 40 mH.

Solution : $i = 0.3\ sin\ (200\ t - 40°)\ A$; $L = 40\ mH = 40\ X\ 10^{-3}\ H$; $V = ?$

♠ In an inductive circuit, the voltage leads the current by 90o. Therefore,

$$v = V_m \sin(\omega\ t + 90°)$$
$$v = I_m\ X_L \sin(\omega\ t + 90°)$$
$$v = I_m\ \omega\ L \sin(\omega\ t + 90°)$$
$$v = 0.3\ X\ 200\ X\ 40\ X\ 10^{-3} \sin(200\ t - 40° + 90°)$$
$$\boldsymbol{v = 2.4 \sin(200\ t + 50°)\ volt}$$

26. A series RLC circuit which resonates at 400 kHz has 80 μH inductor, 2000 pF capacitor and 50 Ω resistor. Calculate (a) Q-factor of the circuit (b) the new value of capacitance when the value of inductance is doubled and (c) the new Q-factor.

Solution : $f_r = 400\ kHz = 400\ X\ 10^3\ Hz$; $L = 80\ \mu H = 80\ X\ 10^{-6}\ H$; $C = 2000\ pF = 2000\ X\ 10^{-12}\ F$; $R = 50\ \Omega$

(a) Q-factor of the circuit,

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{50}\ X \sqrt{\frac{80\ X\ 10^{-6}}{2000\ X\ 10^{-12}}} = \frac{1}{50}\ X \sqrt{\frac{80\ X\ 10^6}{2000}} = \frac{1}{50}\ X \sqrt{\frac{8\ X\ 10^7}{2\ X 10^3}}$$
$$Q = \frac{1}{50}\ X \sqrt{4\ X\ 10^4} = \frac{2\ X\ 10^2}{50} = \frac{200}{50} = \mathbf{4}$$

(b) Resonance frequency ;

$$f_r = \frac{1}{2\ \pi \sqrt{L\ C}} \qquad (or) \qquad {f_r}^2 = \frac{1}{4\ \pi^2\ L\ C}$$
$$\therefore \quad C = \frac{1}{4\ \pi^2\ L\ {f_r}^2}$$

When inductance L is doubled, new capacitance ,

$$C_{new} = \frac{1}{4\ \pi^2\ (2\ L)\ {f_r}^2}$$
$$C_{new} = \frac{1}{4\ X\ (3.14)^2\ X\ 2\ X\ 80\ X\ 10^{-6}\ X\ (400\ X\ 10^3)^2}$$
$$C_{new} = \frac{1}{4\ X\ 3.14\ X\ 3.14\ X\ 2\ X\ 80\ X\ 160000}$$
$$C_{new} = \frac{1}{3.14\ X\ 3.14\ X\ 640\ X\ 160000}$$
$$C_{new} = 9.9\ 06\ X\ 10^{-10}$$
$$C_{new} \cong 10\ X\ 10^{-10} = 1000\ X\ 10^{-12}\ F$$
$$\boldsymbol{C_{new} \cong 1000\ pF}$$

No	Log
Nr	
1	0. 0000
Dr	
3.14	0. 4969
3.14	0. 4969
640	2. 8062
160000	5. 2041
(+)	9. 0041
Nr	0. 0000
Dr	9. 0041
(-)	$\overline{10}.\ 9959$
ALog	$9.\ 906\ X\ 10^{-10}$

(c) New Q- factor,

$$Q_{new} = \frac{1}{R}\sqrt{\frac{2L}{C_{new}}} = \frac{1}{50}\ X \sqrt{\frac{2\ X\ 80\ X\ 10^{-6}}{1000\ X\ 10^{-12}}}$$
$$Q_{new} = \frac{1}{50}\ X \sqrt{\frac{160\ X\ 10^6}{1000}} = \frac{1}{50}\ X \sqrt{\frac{16\ X\ 10^7}{1\ X 10^3}} = \frac{1}{50}\ X \sqrt{16\ X\ 10^4}$$
$$Q_{new} = \frac{4\ X\ 10^2}{50} = \frac{400}{50} = \mathbf{8}$$

27. capacitor of capacitance $\frac{10^{-4}}{\pi}$ F, an inductor of inductance $\frac{2}{\pi}$ H and a resistor of resistance 100 Ω are connected to form a series *RLC* circuit. When an AC supply of 220 V, 50 Hz is applied to the circuit, determine (a) the impedance of the circuit (b) the peak value of current flowing in the circuit (c) the power factor of the circuit and (d) the power factor of the circuit at resonance.

Solution : $C = \frac{10^{-4}}{\pi}\ F$; $L = \frac{2}{\pi}\ H$; $R = 100\ \Omega$; $V_{rms} = 220\ V$; $f = 50\ Hz$

(a) Inductive reactance,

$$X_L = \omega\ L = 2\ \pi\ f\ L = 2\ X\ \pi\ X\ 50 X\ \frac{2}{\pi} = \mathbf{200\ \Omega}$$

Capacitive reactance,

$$X_C = \frac{1}{\omega\ C} = \frac{1}{2\ \pi\ f\ C} = \frac{1}{2\ \pi\ X\ 50\ X\ \frac{10^{-4}}{\pi}} = \frac{10^4}{100} = \mathbf{100\ \Omega}$$

Impedance ;

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$
$$Z = \sqrt{100^2 + (200 - 100)^2}$$
$$Z = \sqrt{100^2 + 100^2}$$
$$Z = \sqrt{2\ X\ 100^2} = \sqrt{2}\ X\ 100 = 1.414\ X\ 100$$
$$\boldsymbol{Z = 141.4\ \Omega}$$

(b) Peak value of current,

$$I_m = \frac{V_m}{Z} = \frac{V_{rms}\sqrt{2}}{Z}$$
$$I_m = \frac{220\ X\ 1.414}{141.1} = \frac{220}{100}$$
$$\boldsymbol{I_m = 2.2\ A}$$

(c) Power factor of the circuit,

$$\boldsymbol{\cos\phi} = \frac{R}{Z} = \frac{100}{141.4} = \frac{1}{1.414} = \frac{1}{\sqrt{2}} = \mathbf{0.707}$$

(d) Power factor of the circuit at resonance ($Z = R$),

$$\boldsymbol{\cos\phi} = \frac{R}{Z} = \frac{R}{R} = \mathbf{1}$$

EXERCISE PROBLEMS

1. A square coil of side 30 cm with 500 turns is kept in a uniform magnetic field of 0.4 T. The plane of the coil is inclined at an angle of 30° to the field. Calculate the magnetic flux through the coil.

Solution : $N = 500$; $a = 30\ cm$; $A = a^2 = 900\ cm^2 = 900\ X\ 10^{-4}\ m^2$;
$\theta = 90° - 30° = 60°$; $B = 0.4$ T ; $N\ \Phi_B = ?$

- Total Magnetic flux through the coil,

$$N\ \Phi_B = N\ B\ A \cos\theta$$
$$N\ \Phi_B = 500\ X\ 0.4\ X\ 900\ X\ 10^{-4} \cos 60°$$
$$N\ \Phi_B = 180000\ X\ 10^{-4}\ X\ \frac{1}{2} = 18\ X\ \frac{1}{2} = \mathbf{9\ Wb}$$

2. A straight metal wire crosses a magnetic field of flux 4 mWb in a time 0.4 s. Find the magnitude of the emf induced in the wire.

Solution : $d\Phi_B = 4\ mWb = 4\ X\ 10^{-3}\ Wb$; $dt = 0.4\ T$; $\epsilon = ?$

- Magnitude of the emf induced in the wire,

$$\epsilon = \frac{d\ \Phi_B}{dt} = \frac{4\ X\ 10^{-3}}{0.4} = \mathbf{10\ X 10^{-3}\ V = 10\ mV}$$

3. The magnetic flux passing through a coil perpendicular to its plane is a function of time and is given by $\Phi_B = (2\ t^3 + 4\ t^2 + 8\ t + 8)$ Wb. If the resistance of the coil is 5 Ω, determine the induced current through the coil at a time t = 3 second.

Solution :

- Magnitude of the induced emf,

$$\epsilon = \frac{d\ \Phi_B}{dt} = \frac{d}{dt}(2\ t^3 + 4\ t^2 + 8\ t + 8)$$
$$\epsilon = 2\ X\ 3t^2 + 4\ X\ 2\ t + 8 + 0 = 6\ t^2 + 8\ t + 8$$

If t = 3 s , $\epsilon = 6\ (3)^2 + 8\ (3) + 8 = 54 + 24 + 8 = \mathbf{86\ V}$

- Then the induced current through the coil,

$$i = \frac{\epsilon}{R} = \frac{86}{5} = \mathbf{17.2\ A}$$

4. A closely wound circular coil of radius 0.02 m is placed perpendicular to the magnetic field. When the magnetic field is changed from 8000 T to 2000 T in 6 s, an emf of 44 V is induced in it. Calculate the number of turns in the coil.

Solution : $r = 0.02$ m ; dt = 6 s ; $dB = 8000 - 2000 = 6000\ T$; $\epsilon = 44\ V$
$\theta = 90° - 90° = 0°$; $N = ?$

- Magnitude of the induced emf ;

$$\epsilon = N\ \frac{d\ \Phi_B}{dt} = N\ \frac{d}{dt}(B\ A \cos\theta) = N\ A \cos\theta \left(\frac{dB}{dt}\right)$$

$$\therefore\ N = \frac{\epsilon}{A \cos\theta\ \left(\frac{dB}{dt}\right)} = \frac{\epsilon}{\pi\ r^2 \cos\theta\ \left(\frac{dB}{dt}\right)} = \frac{44}{\frac{22}{7}\ X\ (0.02)^2 \cos 0°\ \left(\frac{6000}{6}\right)}$$

$$N = \frac{44\ X\ 7\ X\ 6}{22\ X\ 0.0004\ X\ 1\ X\ 6000} = \frac{84}{2.4} = \frac{840}{24} = 35\ turns$$

5. A rectangular coil of area 6 cm² having 3500 turns is kept in a uniform magnetic field of 0.4 T. Initially, the plane of the coil is perpendicular to the field and is then rotated through an angle of 180°. If the resistance of the coil is 35 Ω, find the amount of charge flowing through the coil.

Solution : $A = 6\ cm^2 = 6\ X\ 10^{-4}\ m^2$; $N = 3500$; $B = 0.4\ T$; $\theta_i = 90° - 90° = 0°$
$\theta_f = 180° - 90° = 90°$; $R = 35\Omega$; $q = ?$

- Initial magnetic flux ; $N\ \Phi_B = N\ B\ A \cos\theta_i = N\ B\ A \cos 0° = N\ B\ A$
- Final magnetic flux ; $N\ \Phi_B = N\ B\ A \cos\theta_f = N\ B\ A \cos 180° = -\ N\ B\ A$
- Change in magnetic flux ; $d\ (N\ \Phi_B) = N\ A\ B - (-\ N\ B\ A) = 2\ N\ B\ A$
- Hence rate of change in magnetic flux (i.e.) induced emf;

$$\epsilon = \frac{d\ (N\ \Phi_B)}{dt} = 2\ N\ B\ A$$
$$\epsilon = 2\ X\ 3500\ X\ 0.4\ X\ 6\ X\ 10^{-4} = 16800\ X\ 10^{-4}$$
$$\epsilon = 168\ X\ 10^{-2}\ V$$

- Thus induced current (rate of flow of electric charge).

$$\boldsymbol{i} = \frac{\epsilon}{R} = \frac{168\ X\ 10^{-2}}{35} = \mathbf{4.8\ X\ 10^{-2}\ A}$$

- So the amount of charge flowing through the coil,

$$q = i\ t = 4.8\ X\ 10^{-2}\ X\ 1 = \mathbf{4.8\ X\ 10^{-2}\ C}$$

6. An induced current of 2.5 mA flows through a single conductor of resistance 100 Ω. Find out the rate at which the magnetic flux is cut by the conductor.

Solution : $R = 100\ \Omega$; $i = 2.5\ mA = 2.5\ X\ 10^{-3}\ A$; $\frac{d\ \Phi_B}{dt} = ?$

- The rate of change in magnetic flux (i.e.) induced emf

$$\frac{\boldsymbol{d\ \Phi_B}}{\boldsymbol{dt}} = \epsilon = i\ R = 2.5\ X\ 10^{-3}\ X\ 100 = 250\ X\ 10^{-3} = \mathbf{250\ m\ Wb\ s^{-1}}$$

7. A fan of metal blades of length 0.4 m rotates normal to a magnetic field of $4\ X\ 10^{-3}\ T$. If the induced emf between the centre and edge of the blade is 0.02 V, determine the rate of rotation of the blade.

Solution : $l = 0.4$ m ; $A = \pi\ l^2 = 0.16\ \pi$; $B = 4\ X\ 10^{-3}\ T$; $\epsilon_m = 0.02\ V$; $\omega = ?$

- Maximum value of induced emf,

$$\epsilon_m = N\ B\ A\ \omega$$
$$\therefore\ \omega = \frac{\epsilon_m}{N\ B\ A} = \frac{0.02}{1\ X\ 4\ X\ 10^{-3}\ X\ 0.16\ \pi}$$
$$\omega = \frac{0.02}{0.64\ X\ 10^{-3}\ X\ 3.14}$$
$$\omega = \frac{0.02}{0.64\ X\ 10^{-3}\ X\ 3.14}$$
$$\omega = \frac{0.02\ X\ 10^3}{2.0096}$$
$$\omega = 9.954\ X\ 10^{-3} X 10^3$$
$$\boldsymbol{\omega = 9.954\ \text{revolutions/second}}$$

No	Log
0.02 2.0096	$\bar{2}.3010$ 0. 3030
(-) ALog	$\bar{3}.9980$ $9.954\ X\ 10^{-3}$

8. A bicycle wheel with metal spokes of 1 m long rotates in Earth's magnetic field. The plane of the wheel is perpendicular to the horizontal component of Earth's field of $4 X 10^{-5} T$. If the emf induced across the spokes is 31.4 mV, calculate the rate of revolution of the wheel.

Solution : $l = 1\, m$; $B_H = 4 X 10^{-5} T$; $\epsilon = 31.4\, mV = 31.4 X 10^{-3} V$; $\omega = ?$

♠ Maximum value of induced emf,

$$\epsilon_m = N B A \omega = N B (\pi l^2)\omega$$

$$\therefore \quad \omega = \frac{\epsilon_m}{N B \pi l^2} = \frac{31.4 X 10^{-3}}{1 X 4 X 10^{-5} X 3.14 X 1^2}$$

$$\omega = \frac{10 X 10^2}{4} = \frac{1000}{4}$$

$$\boldsymbol{\omega = 250 \text{ revolutions/second}}$$

9. Determine the self-inductance of 4000 turn air-core solenoid of length 2m and diameter 0.04 m.

Solution : : $l = 2\, m$; $d = 0.04\, m$; $r = 0.02\, m$; $N = 4000$; $L = ?$

♠ Self inductance of air core solenoid,

$$L = \frac{\mu_o N^2 A}{l} == \frac{\mu_o N^2 \pi r^2}{l}$$

$$L = \frac{4 \pi X 10^{-7} X 4000^2 X \pi X 0.02 X 0.02}{2}$$

$$L = 2 X 3.14 X 3.14 X 10^{-7} X 16 X 10^6 X 0.02 X 0.02$$

$$L = 2 X 3.14 X 3.14 X 16 X 4 X 10^{-5}$$

$$L = 128 X 3.14 X 3.14 X 10^{-5}$$

$$L = 1.262 X 10^3 X 10^{-5} = 1.262 X 10^{-2} H = 12.62 X 10^{-3} H$$

$$\boldsymbol{L = 12.62\, mH}$$

No	Log
128	2.1072
3.14	0.4969
3.14	0.4969
(+)	3.1010
ALog	$1.262 X 10^3$

10. A coil of 200 turns carries a current of 4 A. If the magnetic flux through the coil is 6×10^{-5} Wb, find the magnetic energy stored in the medium surrounding the coil.

Solution : $N = 200$; $I = 4\, A$; $\Phi_B = 6 X 10^{-5} Wb$; $U_B = ?$

♠ Magnetic energy stored,

$$U_B = \frac{1}{2} L I^2 = \frac{1}{2}\left[\frac{N \Phi_B}{I}\right] I^2 = \frac{1}{2} N \Phi_B I$$

$$U_B = \frac{1}{2} X 200 X 6 X 10^{-5} X 4$$

$$U_B = 100 X 6 X 10^{-5} X 4 = 2400 X 10^{-5} J$$

$$\boldsymbol{U_B = 0.024\, J}$$

11. A 50 cm long solenoid has 400 turns per cm. The diameter of the solenoid is 0.04 m. Find the magnetic flux linked with each turn when it carries a current of 1 A.

Solution : $l = 50\, cm = 0.5\, m$; $n = 400$; $N = n\, l = 400 X 50 = 20000$; $d = 0.04\, m$; $r = 0.02\, m$; $I = 1\, A$; $\Phi_B = ?$

♠ Let 'L' be the self inductance of the solenoid, magnetic flux is,

$$\Phi_B = L I = \frac{\mu_o N^2 A}{l} I = \frac{\mu_o N^2 \pi r^2}{l} I$$

$$\Phi_B = \frac{4 \pi X 10^{-7}\ 20000^2 X 3.14 X 0.02^2}{0.5} X 1$$

$$\Phi_B = \frac{4 X 3.14 X 10^{-7} X 4 X 10^8 X 3.14 X 0.0004}{0.5} X 1$$

$$\Phi_B = \frac{64 X 3.14 X X 3.14 X 10^{-3}}{0.5}$$

$$\Phi_B = 128 X 3.14 X 3.14 X 10^{-3}$$

$$\Phi_B = 1.262 X 10^3 X 10^{-3} = \boldsymbol{1.262\, Wb}$$

No	Log
128	2.1072
3.14	0.4969
3.14	0.4969
(+)	3.1010
ALog	$1.262 X 10^3$

♠ Hence magnetic flux linked with each turn

$$\frac{\boldsymbol{\Phi_B}}{N} = \frac{1.262}{20000} = \boldsymbol{0.631 X 10^{-4} Wb}$$

12. A coil of 200 turns carries a current of 0.4 A. If the magnetic flux of 4 mWb is linked with each turn of the coil, find the inductance of the coil.

Solution : $N = 200$; $I = 0.4\, A$; $\Phi_B = 4\, mWb = 4 X 10^{-3} Wb$; $L = ?$

♠ Self inductance of the coil,

$$L = \frac{N \Phi_B}{I} = \frac{200 X 4 X 10^{-3}}{0.4} = 2000 X 10^{-3} = \boldsymbol{2\, H}$$

13. Two air core solenoids have the same length of 80 cm and same cross–sectional area 5 cm². Find the mutual inductance between them if the number of turns in the first coil is 1200 turns and that in the second coil is 400 turns.

Solution : $l = 80\, cm = 80 X 10^{-2}\, m$; $A = 5\, cm^2 = 5 X 10^{-4}\, m^2$; $N_1 = 1200$; $N_2 = 400$; $M = ?$

♠ Mutual inductance between the coils,

$$M = \frac{\mu_o N_1 N_2 A}{l} = \frac{4 \pi X 10^{-7} X 1200 X 400 X 5 X 10^{-4}}{80 X 10^{-2}}$$

$$M = 4 \pi X 15 X 400 X 5 X 10^{-9} = 3.14 X 12 X 10^{-5}$$

$$M = 37.68 X 10^{-5} H = 0.3768 X 10^{-3} H = \boldsymbol{0.3768\, mH}$$

14. A long solenoid having 400 turns per cm carries a current 2A. A 100 turn coil of cross-sectional area 4 cm² is placed co-axially inside the solenoid so that the coil is in the field produced by the solenoid. Find the emf induced in the coil if the current through the solenoid reverses its direction in 0.04 sec.

Solution :: $n_1 = 400$; $N_1 = 400 X 100 = 4 X 10^4$; $N_2 = 100$; $A = 4\, cm^2 = 4 X 10^{-4}\, m^2$; $I_1 = 2\, A$; $t = 0.04\, s$

♠ Mutual inductance,

$$M = \frac{\mu_o N_1 N_2 A}{l} = \frac{4 \pi X 10^{-7} X 4 X 10^4 X 100 X 4 X 10^{-4}}{1}$$

$$M = 64 \pi X 10^{-5} = 64 X 3.14 X 10^{-5}$$

$$M = 200.96 X 10^{-5} = 2.0096 X 10^{-3}$$

$$\boldsymbol{M \cong 2 X 10^{-3}\, H}$$

♠ If the current through the solenoid is reversed its direction, the emf inuced in the coil is,

$$\epsilon_2 = M \frac{dI_1}{dt} = 2 X 10^{-7} X \frac{2-(-2)}{0.04} = 2 X 10^{-3} X \frac{4}{0.04}$$

$$\epsilon_2 = 2 X 10^{-3} X 100 = 2 X 10^{-1} V = \boldsymbol{0.2\, V}$$

15. A 200 turn circular coil of radius 2 cm is placed co-axially within a long solenoid of 3 cm radius. If the turn density of the solenoid is 90 turns per cm, then calculate mutual inductance of the coil and the solenoid.

Solution : $r = 2\ cm = 2\ X\ 10^{-2}\ m$; $N_1 = 200$; $N_2 = 90\ X 100 = 9000$

- Mutual inductance of the coil and the solenoid,

$$M = \frac{\mu_o N_1 N_2 A}{l} = \frac{\mu_o N_1 N_2 \pi r^2}{l}$$

$$M = \frac{4\pi\ X\ 10^{-7} X\ 200\ X\ 9000\ X\ 3.14\ \ X\ (2\ X\ 10^{-2})^2}{1}$$

$$M = 3.14\ X\ 3.14\ X\ 288\ X\ 10^{-6} = 2.839\ X\ 10^3\ X\ 10^{-6}$$

$$\boldsymbol{M = 2.839\ X\ 10^{-3}\ H = 2.839\ m\ H}$$

No	Log
3.14	0. 4969
3.14	0. 4969
288	2. 4594
(+) ALog	3. 4532 $2.839\ X\ 10^3$

16. The solenoids S_1 and S_2 are wound on an iron-core of relative permeability 900. Their areas of their cross-section and their lengths are the same and are 4 cm² and 0.04 m respectively. If the number of turns in S_1 is 200 and that in S_2 is 800, calculate the mutual inductance between the solenoids. If the current in solenoid 1 is increased form 2A to 8A in 0.04 second, calculate the induced emf in solenoid 2.

Solution : $\mu_r = 900$; $A = 4\ cm^2 = 4\ X\ 10^{-4}\ m^2$; $l = 0.04\ m$; $N_1 = 200$; $N_2 = 800$; $dI_1 = 8 - 2 = 6\ A$; $dt = 0.04\ s$; $M = ?$; $\epsilon_2 = ?$

- Mutual inductance,

$$M = \frac{\mu_o\ \mu_r\ N_1 N_2\ A}{l} = \frac{4\ \pi\ X\ 10^{-7} X\ 900\ X\ 200\ X\ 800\ X\ 4\ X\ 10^{-4}}{0.04}$$

$$M = 100\ \pi\ X\ 10^{-7} X\ 900\ X\ 200\ X\ 800\ X\ 4\ X\ 10^{-4}$$

$$M = 3.14\ X\ 576\ X\ 10^{-3}$$

$$\boldsymbol{M} = 1.808\ X\ 10^3\ X\ 10^{-3} \cong \boldsymbol{1.81\ H}$$

No	Log
3.14	0. 4969
576	2. 7604
(+) ALog	3. 2573 $1.808\ X\ 10^3$

- The induced emf in solenoid 2 is,

$$\epsilon_2 = -\ M\ \frac{dI_1}{dt} = -1.81\ X \frac{6}{0.04} = \ -1.81\ X \frac{600}{4}$$

$$\epsilon_2 = -1.81\ X\ 150\ = \boldsymbol{-271.5\ V}$$

17. A step-down transformer connected to main supply of 220 V is used to operate 11V, 88 W lamp. Calculate (a) Voltage transformation ratio and (b) Current in the primary.

Solution : $V_P = 220\ V$; $V_S = 11\ V$; $P_S = 88\ W$; $K = ?$; $I_P = ?$

(a) Voltage transformation ratio ; $K = \frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$

$$\therefore\ \boldsymbol{K} = \frac{V_S}{V_P} = \frac{11}{220} = \boldsymbol{\frac{1}{20}}$$

(b) We know the output power ; $P_S = 88\ W$

$(i.e)$ $V_S\ I_S\ =\ 88\ W$

Hence, the secondary current ;

$$I_S\ =\ \frac{88}{V_S} = \frac{88}{11} = 8\ A$$

- Thus, current in the primary

$$I_P = K\ I_S = \frac{1}{20}\ X\ 8 = \frac{4}{10} = \boldsymbol{0.4\ A}$$

18. A 200V/120V step-down transformer of 90% efficiency is connected to an induction stove of resistance 40 Ω. Find the current drawn by the primary of the transformer.

Solution : $\eta = 90\ \%$; $V_P = 200\ V$; $V_S = 120\ V$; $R = 40\ \Omega$

- Output power ; $P_S = V_S\ I_S = V_S\ \frac{V_S}{R} = \frac{V_S^2}{R} = \frac{120^2}{40} = \frac{120\ X\ 120}{40} = 360\ W$
- Thus secondary current is; $I_S = \frac{P_S}{V_S} = \frac{360}{120} = \boldsymbol{3\ A}$
- We know, the efficiency of the transformer ; $\eta = \frac{P_S}{P_P} = \frac{V_S\ I_S}{V_P\ I_P}$

$$\frac{90}{100} = \frac{360}{200\ X\ I_P}$$

- The current drawn by the primary of the transformer

$$I_P = \frac{360\ X\ 100}{200\ X\ 90} = \frac{36000}{18000} = \boldsymbol{2\ A}$$

19. The 300 turn primary of a transformer has resistance 0.82 Ω and the resistance of its secondary of 1200 turns is 6.2 Ω. Find the voltage across the primary if the power output from the secondary at 1600V is 32 kW. Calculate the power losses in both coils when the transformer efficiency is 80%.

Solution : $N_P = 300$; $N_S = 1200$; $R_P = 0.82\ \Omega$; $R_S = 6.2\ \Omega$; $V_S = 1600\ V$

$$P_S = 32\ kW = 32\ X\ 10^3\ W \quad ; \quad \eta = 80\ \% = \frac{80}{100}$$

- Output power ; $P_S = V_S\ I_S$ (or) $\boldsymbol{I_S} = \frac{P_S}{V_S} = \frac{32\ X\ 10^3}{1600} = \frac{32000}{1600} = \boldsymbol{20\ A}$
- Transformer equation ; $\frac{V_S}{V_P} = \frac{N_S}{N_P}$ (or) $\boldsymbol{V_P} = \frac{V_S\ N_P}{N_S} = \frac{1600\ X\ 300}{1200} = \frac{4800}{12} = \boldsymbol{400\ V}$
- Efficiency ; $\eta = \frac{P_S}{P_P} = \frac{P_S}{V_P\ I_P}$

$$\frac{80}{100} = \frac{32\ X\ 10^3}{400\ X\ I_P}$$

$$\boldsymbol{I_P} = \frac{32\ X\ 10^3\ X\ 100}{400\ X\ 80} = \frac{3200}{32} = \boldsymbol{100\ A}$$

- Power loss in primary coil $= I_P^2\ R_P\ = 100^2\ X\ 0.82 = 8200\ W\ = \boldsymbol{8.2\ kW}$
- Power loss in secondary coil $= I_S^2\ R_S = 20^2\ X\ 6.2\ = 2480\ W\ = \boldsymbol{2.48\ kW}$

20. Calculate the instantaneous value at 60º, average value and RMS value of an alternating current whose peak value is 20 A.

Solution : $I_m = 20\ A$; $\omega t = 60°$; $i = ?$; $I_{avg} = ?$; $I_{rms} = ?$

- Alternating current at any instant,

$$i = I_m \sin \omega t\ = 20 \sin 60° = 20\ X\ \frac{\sqrt{3}}{2} = 10\ X\ 1.732\ = \boldsymbol{17.32\ A}$$

- Average value of alternating current,

$$I_{avg} = \frac{2\ I_m}{\pi} = 0.637\ I_m = 0.637\ X\ 20 = \ \boldsymbol{12.74\ A}$$

- **RMS** value of alternating current,

$$I_{rms} = \frac{I_m}{\sqrt{2}} = 0.707\ I_m = 0.707\ X\ 20\ = \ \boldsymbol{14.14\ A}$$