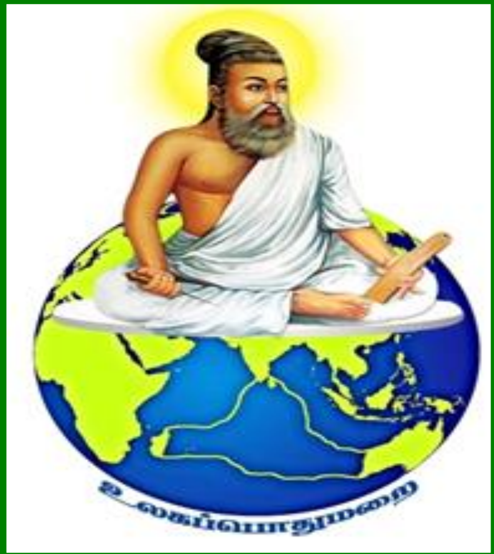
HIGHER SECONDARY
SECOND YEAR

PHYSICS



UNIT -3

MAGNETISM
AND
MAGNETIC EFFECTS OF ELECTRIC CURRENT

PROBLEMS AND SOLUTIONS

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EXAMPLE PROBLEMS

1. The horizontal component and vertical component of Earth's magnetic field at a place are 0.15 G and 0.26 G respectively. Calculate the angle of dip and resultant magnetic field. (G-gauss, cgs unit for magnetic field 1G = 10−4 T)

Solution : $B_H = 0.15\ G$; $B_V = 0.26\ G$; $I = ?$; $B = ?$

- ♣ Angle of dip 'I' is given by,

$$\tan I = \frac{B_V}{B_H} = \frac{0.26}{0.15} = \frac{26}{15} = 1.733$$

$$\boldsymbol{I} = \tan^{-1}(1.733) = \mathbf{60^\circ}$$

- ♣ Resultant magnetic field,

$$B = \sqrt{{B_H}^2 + {B_V}^2} = \sqrt{0.15^2 + 0.26^2}$$

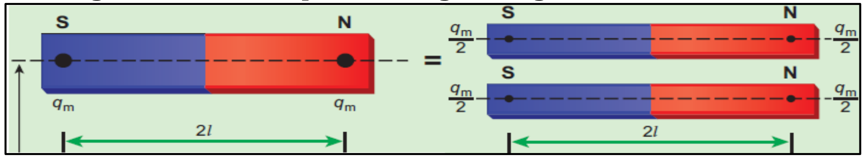
$$B = \sqrt{0.0225 + 0.0676} = \sqrt{0.0901}$$

$$\boldsymbol{B = 0.3\ G}$$

2. Let the magnetic moment of a bar magnet be $\vec{p}_m$ whose magnetic length is $d = 2l$ and pole strength is q_m. Compute the magnetic moment of the bar magnet when it is cut into two pieces (a) along its length (b) perpendicular to its length.

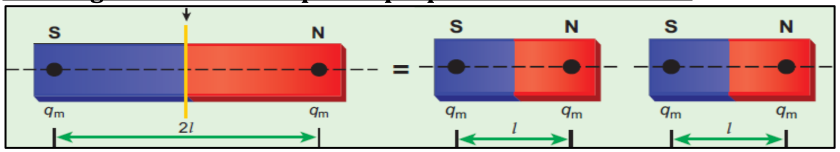
Solution :

(a) <u>Bar magnet cut into two pieces along its length</u> :



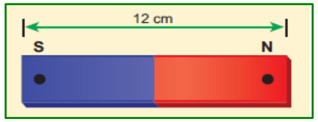
- ♣ When the bar magnet is cut along the axis into two pieces, the magnetic length does change , but magnetic pole strength becomes halved . So the new magnetic pole strength is ; $q_m' = \frac{q_m}{2}$
- ♣ Hence the magnetic moment ; $p_m' = q_m' d = \frac{1}{2} q_m\ 2\ l = \frac{1}{2}\ p_m$
- ♣ In vector notation ; $\vec{p}_m' = \frac{1}{2}\ \vec{p}_m$

(b) <u>Bar magnet cut into two pieces perpendicular to the axis</u> :



- ♣ When the bar magnet is cut perpendicular to the axis into two pieces, the magnetic pole strength does change , but magnetic length becomes halved. So the new magneticlength is ;: $d' = \frac{d}{2} = l$
- ♣ Hence magnetic moment ; $p_m' = q_m\ d' = q_m\ \frac{1}{2} d = \frac{1}{2}\ q_m\ (2\ l) = \frac{1}{2}\ p_m$
- ♣ In vector notation ; $\vec{p}_m' = \frac{1}{2}\ \vec{p}_m$

3. Compute the magnetic length of a uniform bar magnet if the geometrical length of the magnet is 12 cm. Mark the positions of magnetic pole points.



Solution :

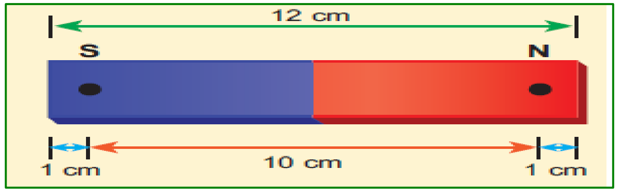
- ♣ We know that, Magnetic length : Geometrical length = 5 : 6

(i.e.) $$\frac{Magnetic\ length}{Geometrical\ length} = \frac{5}{6}$$

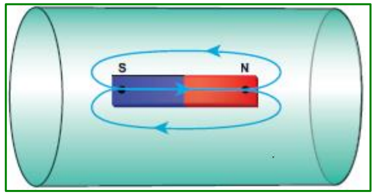
$$Magnetic\ length = \frac{5}{6}\ X\ Geometrical\ length = \frac{5}{6}\ X\ 12$$

$$Magnetic\ length = 10\ cm$$

- ♣ In this figure, the dot implies the pole points.



4. Calculate the magnetic flux coming out from closed surface containing magnetic dipole (say, a bar magnet) as shown in figure.



Solution :

- ♣ The total flux emanating from the closed surface S enclosing the dipole is zero. So,

$$\boldsymbol{\Phi_B = \oint \vec{B}\,.\,\overrightarrow{dA} = 0}$$

- ♣ Here the integral is taken over closed surface. Since no isolated magnetic pole (called magnetic monopole) exists, this integral is always zero. This is similar to Gauss's law in electrostatics.

5. The repulsive force between two magnetic poles in air is 9 × 10⁻³ N. If the two poles are equal in strength and are separated by a distance of 10 cm, calculate the pole strength of each pole.

Solution : $F = 9\ X\ 10^{-3}\ N$; $r = 10\ cm = 10\ X\ 10^{-2}\ m$; $q_{m_A} = q_{m_B} = q_m = ?$

- ♣ The magnitude of the force between two poles is given by

$$F = \frac{\mu_o}{4\ \pi} \frac{q_{m_A}\ q_{m_B}}{r^2}$$

$$9\ X\ 10^{-3} = \frac{4\ \pi\ X\ 10^{-7}}{4\ \pi} \frac{{q_m}^2}{(10\ X\ 10^{-2})^2}$$

$$9\ X\ 10^{-3} = 10^{-7} X\ \frac{{q_m}^2}{10^{-2}}$$

$${q_m}^2 = \frac{9\ X\ 10^{-3}}{10^{-5}} = 900$$

$$\therefore\ \boldsymbol{q_m = 30\ N\ T^{-1}}$$

6. A short bar magnet has a magnetic moment of 0.5 J T^{-1}. Calculate magnitude and direction of the magnetic field produced by the bar magnet which is kept at a distance of 0.1 m from the centre of the bar magnet along (a) axial line of the bar magnet and (b) normal bisector of the bar magnet.

Solution : $p_m = 0.5\, J\, T^{-1}$; $r = 0.1\, m$; $B_{axis} = ?$; $B_{equator} = ?$

(a) Magnetic field at a point on the axial line of the bar magnet,

$$\vec{B}_{axis} = \frac{\mu_o}{4\,\pi}\frac{2\,p_m}{r^3}\,\hat{\imath}$$

$$\vec{B}_{axis} = \frac{4\,\pi\,X\,10^{-7}}{4\,\pi}\frac{2\,X\,0.5}{(0.1)^3}\,\hat{\imath}$$

$$\vec{B}_{axis} = 10^{-7}\,X\,\frac{1}{0.001}\hat{\imath} = 10^{-7}\,X\,\frac{1}{10^{-3}}\,\hat{\imath}$$

$$\vec{B}_{axis} = \mathbf{1\,X\,10^{-4}}\,\hat{\imath}$$

- Hence, the magnitude of the magnetic field along axial is $\mathbf{B_{axis} = 1 \times 10^{-4}\,T}$ and direction is towards South to North.

(b) Magnetic field at a point on the normal bisector of the bar magnet,

$$\vec{B}_{equator} = -\frac{\mu_o}{4\,\pi}\frac{p_m}{r^3}\,\hat{\imath}$$

$$\vec{B}_{equator} = -\frac{4\,\pi\,X\,10^{-7}}{4\,\pi}\frac{0.5}{(0.1)^3}\,\hat{\imath}$$

$$\vec{B}_{equator} = -10^{-7}\,X\,\frac{0.5}{0.001}\hat{\imath} = 10^{-7}\,X\,\frac{0.5}{10^{-3}}\,\hat{\imath}$$

$$\vec{B}_{equator} = -\mathbf{0.5\,X 10^{-4}}\,\hat{\imath}$$

- Hence, the magnitude of the magnetic field along equatorial is $B_{equator} = 0.5 \times 10^{-4}$ T and direction is towards North to South.

Note : The magnitude of B_{axis} is twice that of magnitude of $B_{equator}$ and the direction of B_{axis} and $B_{equator}$ are opposite.

7. Consider a magnetic dipole which on switching ON external magnetic field orient only in two possible ways i.e., one along the direction of the magnetic field (parallel to the field) and another anti-parallel to magnetic field. Compute the energy for the possible orientation.

Solution :

- Let $\boldsymbol{p_m}$ be the dipole and before switching ON the external magnetic field, there is no orientation. Therefore, the energy U = 0.
- As soon as external magnetic field is switched ON, the magnetic dipole orient parallel (θ = 0°) to the magnetic field with energy,

$$U_{parallel} = -\,p_m\,B\,\cos 0^o = -\,p_m\,B = minimum$$

- Otherwise, the magnetic dipole orients anti-parallel (θ = 180°) to the magnetic field with energy,

$$U_{anti\,parallel} = -\,p_m\,B\,\cos 180^o = +\,p_m\,B = maximum$$

8. Compute the intensity of magnetisation of the bar magnet whose mass, magnetic moment and density are 200 g, 2 A m^2 and 8 g cm^{-3}, respectively.

Solution : $m = 200g = 200\,X10^{-3}\,kg\,; p_m = 2\,Am^2; \rho = 8\,gcm^{-3} = 8\,X10^3 kg\,m^{-3}$

- Density of the bar magnet ;

$$\rho = \frac{mass}{Volume} = \frac{m}{V}$$

- Hence the volume ;

$$V = \frac{m}{\rho} = \frac{200\,X10^{-3}}{8\,X10^3} = 25\,X\,10^{-6}\,m^3$$

- So the intensity of magnetization ;

$$M = \frac{p_m}{V} = \frac{2}{25\,X\,10^{-6}} = \frac{0.08}{10^{-6}} = 0.08\,X\,10^6\,A\,m^{-1}$$

$$\mathbf{M = 8\,X\,10^4\,A\,m^{-1}}$$

9. Using the relation $\vec{B} = \mu_o(\vec{H} + \vec{M})$, show that $\chi_m = \mu_r - 1$.

Solution :

- By definition, the magnetic susceptibility is ; $\chi_m = \frac{\vec{M}}{\vec{H}}$

$$(or) \quad \vec{M} = \chi_m\,\vec{H} \quad ------ \quad (1)$$

- By definition, the magnetic field is ; $\vec{B} = \mu\,\vec{H}$ ------ (2)
- The given relation, $\vec{B} = \mu_o(\vec{H} + \vec{M})$
- put equation (1) and (2), we get

$$\mu\,\vec{H} = \mu_o(\vec{H} + \chi_m\,\vec{H})$$

$$\mu\,\vec{H} = \mu_o\vec{H}\,(1 + \chi_m)$$

$$\mu = \mu_o\,(1 + \chi_m)$$

$$\frac{\mu}{\mu_o} = (1 + \chi_m)$$

$$(or) \quad \mu_r = 1 + \chi_m$$

$$(or) \quad \chi_m = \mu_r - 1$$

10. Two materials X and Y are magnetised whose values of intensity of magnetisation are 500 A m^{-1} and 2000 A m^{-1} respectively. If the magnetising field is 1000 A m^{-1}, then which one among these materials can be easily magnetized?

Solution :

- Susceptibility of material X,

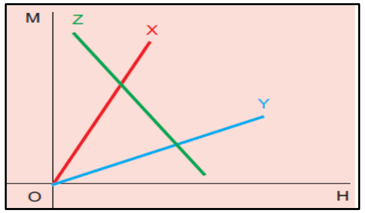
$$\chi_{m_X} = \frac{|\vec{M}|}{|\vec{H}|} = \frac{500}{1000} = \frac{1}{2} = 0.5$$

- Susceptibility of material Y

$$\chi_{m_Y} = \frac{|\vec{M}|}{|\vec{H}|} = \frac{2000}{1000} = 2$$

- Here $\chi_{m_X} < \chi_{m_Y}$ .. It implies that material Y can be easily magnetized.

11. The following figure shows the variation of intensity of magnetisation with the applied magnetic field intensity for three magnetic materials X, Y and Z. Identify the materials X, Y and Z.



Solution :

♣ The slope of *M-H* graph is a measure of the magnetic susceptibility, which is given by

$$Slope = \frac{|\vec{M}|}{|\vec{H}|} = \chi_m$$

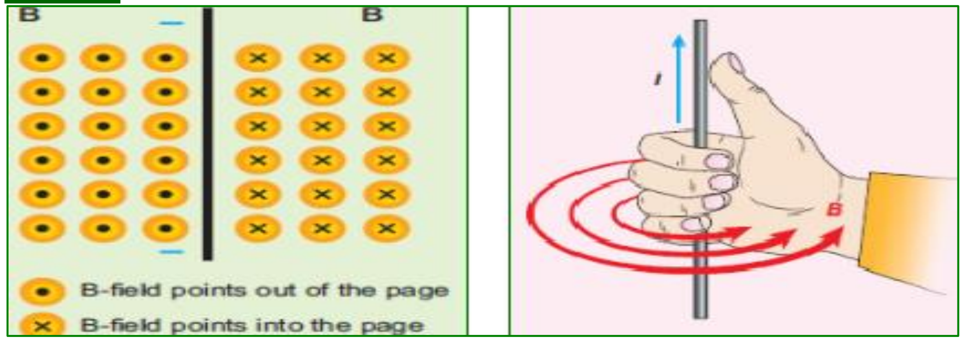
(a) **Material X** : Slope is positive and larger value. So, it is a ferromagnetic material.

(b) **Material Y** : Slope is positive and lesser value than X. So, it could be a paramagnetic material.

(c) **Material Z** : Slope is negative and hence, it is a diamagnetic material.

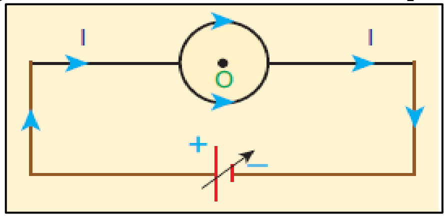
12. The magnetic field shown in the figure is due to the current carrying wire. In which direction does the current flow in the wire?.

Solution :



♣ Using right hand rule, current flows upwards.

13. What is the magnetic field at the centre of the loop shown in figure?



Solution :

♣ The magnetic field due to current in the upper semicircle and lower semicircle of the circular coil are equal in magnitude but opposite in direction.

♣ Hence, the net magnetic field at the centre of the loop (at point O) is zero (i.e.) $\vec{B} = \vec{0}$.

14. A coil of a tangent galvanometer of diameter 0.24 m has 100 turns. If the horizontal component of Earth's magnetic field is 25 × 10⁻⁶ T then, calculate the current which gives a deflection of 60°.

Solution : $N = 100$; $d = 0.24\ m$; $r = 0.12\ m$; $B_H = 25\ X\ 10^{-6}\ T$; $\theta = 60°$

♣ From the theory of TG, the current through the circular coil is,

$$I = \frac{2\ r\ B_H}{\mu_o\ N} \tan\theta$$

$$I = \frac{2\ X\ 0.12\ X\ 25\ X\ 10^{-6}}{4\ \pi\ X\ 10^{-7}\ X\ 100} \tan 60°$$

$$I = \frac{6\ X\ 10^{-6}}{4\ X\ 3.14\ X\ 10^{-5}}\ X\ \sqrt{3}$$

$$I = \frac{6\ X\ 1.732\ X\ 10^{-1}}{12.56}$$

$$I = 8.274\ X\ 10^{-1}\ X\ 10^{-1} = 8.274\ X\ 10^{-2}\ A$$

$$\boldsymbol{I = 0.08274\ A}$$

No	Log
6 1.732	0.7782 0.2385
(+) 12.56	1.0167 1.0990
(-) ALog	$\bar{1}.9177$ $8.274\ X\ 10^{-1}$

15. Compute the magnitude of the magnetic field of a long, straight wire carrying a current of 1 A at distance of 1m from it. Compare it with Earth's magnetic field.

Solution : $I = 1\ A$; $a = 1\ m$; $B = ?$; $B : B_{earth} = ?$

♣ Magnetic field due to long straight current carrying wire,

$$B = \frac{\mu_o\ I}{2\ \pi\ a} = \frac{4\ \pi\ X\ 10^{-7}\ X\ 1}{2\ \pi\ X\ 1} = 2\ X\ 10^{-7}\ T$$

♣ But the Earth's magnetic field is ; $B_{earth} \approx 10^{-5}\ T$

♣ Thus, $\boldsymbol{B : B_{earth} = 1 : 100}$

(i.e.) $B_{straightwire}$ is one hundred times smaller than B_{Earth}

16. Calculate the magnetic field inside a solenoid, when (a) the length of the solenoid becomes twice with fixed number of turns (b) both the length of the solenoid and number of turns are doubled (c) the number of turns becomes twice for the fixed length of the solenoid

Solution :

♣ The magnetic field inside a solenoid of length L and turns N is

$$B_{L,N} = \frac{\mu_o\ N\ I}{L} \qquad ------ \quad (1)$$

(a) If $L \rightarrow 2L$ then ; $B_{2L,N} = \frac{\mu_o\ N\ I}{2\ L} = \frac{B_{L,N}}{2}$

(b) If $L \rightarrow 2L,\ N \rightarrow 2N$ then ; $B_{2L,2N} = \frac{\mu_o\ 2\ N\ I}{2\ L} = \frac{\mu_o\ N\ I}{L} = B_{L,N}$

(c) If $N \rightarrow 2N$ then ; $B_{L,2N} = \frac{\mu_o\ 2\ N\ I}{L} = 2\ B_{L,N}$

♣ Here $\boldsymbol{B_{L,2N} > B_{2L,2N} > B_{2L,N}}$. Thus, strength of the magnetic field is increased when we pack more loops into the same length for a given current

17. A particle of charge q moves with velocity $\vec{v}$ along positive y - direction in a magnetic field $\vec{B}$. Compute the Lorentz force experienced by the particle (a) when magnetic field is along positive y-direction (b) when magnetic field points in positive z - direction (c) when magnetic field is in zy - plane and making an angle θ with velocity of the particle. Mark the direction of magnetic force in each case..

Solution : $\vec{v} = v\,\hat{j}$

(a) When magnetic field is along positive y - direction, then $\vec{B} = B\,\hat{j}$

Lorentz force ; $\vec{F}_m = q\,(\vec{v}\,X\,\vec{B}) = q\,(v\,\hat{j}\,X\,B\,\hat{j}) = q\,v\,B\,(\hat{j}\,X\,\hat{j}) = 0$

So, no force acts on the particle when it moves along the direction of magnetic field.

(b) When magnetic field is along positive z - direction, then $\vec{B} = B\,\hat{k}$

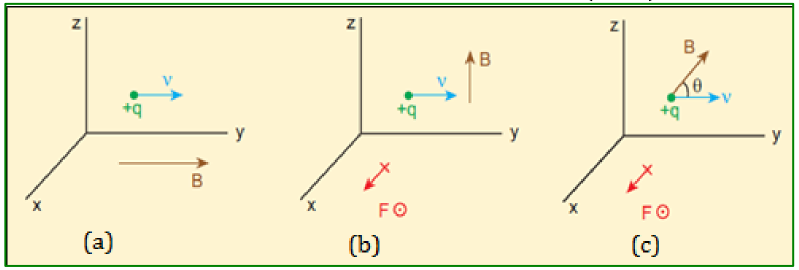
Lorentz force ; $\vec{F}_m = q\,(\vec{v}\,X\,\vec{B}) = q\,(v\,\hat{j}\,X\,B\,\hat{k}) = q\,v\,B\,(\hat{j}\,X\,\hat{k}) = q\,v\,B\,\hat{i}$

Therefore, the magnitude of the Lorentz force is qvB and direction is along positive x - direction.

(c) When magnetic field is in zy -plane, then $\vec{B} = B\cos\theta\,\hat{j} + B\sin\theta\,\hat{k}$

Lorentz force ; $\vec{F}_m = q\,(\vec{v}\,X\,\vec{B}) = q\,(v\,\hat{j}\,X\,B[\cos\theta\,\hat{j} + B\sin\theta\,\hat{k}])$

$$\vec{F}_m = q\,v\,B\,\cos\theta\,(\hat{j}\,X\,\hat{j}) + q\,v\,B\,\sin\theta\,(\hat{j}\,X\,\hat{k}) = q\,v\,B\,\sin\theta\,i$$



18. Compute the work done and power delivered by the Lorentz force on the particle of charge q moving with velocity $\vec{v}$. Calculate the angle between Lorentz force and velocity of the charged particle and also interpret the result.

Solution :

- ♣ Lorentz force on a charged particle moving on a magnetic field is,
$$\vec{F}_m = q\,(\vec{v}\,X\,\vec{B})$$
- ♣ Work done by the magnetic field ; $W = \int \vec{F}\,.\,\vec{dr} = q\int(\vec{v}\,X\,\vec{B})\,.\,\vec{v}\,dt = 0$
- ♣ And power is given by ; $P = \frac{dW}{dt} = 0$
- ♣ Since, $\vec{F}\,.\,\vec{v} = 0$ we have $\vec{F} \perp \vec{v}$. The angle between Lorentz force and velocity of the charged particle is 90º.
- ♣ Thus Lorentz force changes the direction of the velocity but not the magnitude of the velocity. Hence Lorentz force does no work and also does not alter kinetic energy of the particle.

19. An electron moving perpendicular to a uniform magnetic field 0.500 T undergoes circular motion of radius 2.50 mm. What is the speed of electron?

Solution : $B = 0.500\,T$; $r = 2.5\,mm = 2.5\,X\,10^{-3}\,m$; $|e| = 1.6\,X\,10^{-19}\,C$; $v = ?$

- ♣ Lorentz force acts as centripetal force for the particle causing it to execute circular motion. The radius of the circular path is ; $r = \frac{m\,v}{B\,e}$
- ♣ Therefore the speed of the electron ; $v = \frac{B\,e\,r}{m}$

$$v = \frac{0.5\,X\,1.6\,X\,10^{-19}\,X\,2.5\,X\,10^{-3}}{9.1\,X\,10^{-31}}$$

$$v = \frac{0.5\,X\,1.6\,\,X\,2.5\,X\,10^{9}}{9.1}$$

$$v = 2.197\,X\,10^{-1}X\,10^{9}$$

$$\boldsymbol{v = 2.197\,X\,10^{8}\,\,m\,s^{-1}}$$

No	Log
0.5	$\bar{1}$.6990
1.6	0.2041
2.5	0.3979
(+)	0.3010
9.1	0.9590
(-)	$\bar{1}$.3420
ALog	$2.197\,X\,10^{-1}$

20. A proton moves in a uniform magnetic field of strength 0.500 T magnetic field is directed along the x-axis. At initial time, $t = 0$ s, the proton has velocity $\vec{v} = [1.95\,X\,10^5\,\hat{i} + 2,00\,X\,10^5\,\hat{k}]\,m\,s^{-1}$ .Find

(a) At initial time, what is the acceleration of the proton?

(b) Is the path circular or helical? If helical, calculate the radius of helical trajectory and also calculate the pitch of the helix (Note: Pitch of the helix is the distance travelled along the helix axis per revolution).

Solution : $\vec{B} = 0.500\,\hat{i}$; $\vec{v} = [1.95\,X\,10^5\,\hat{i} + 2.00\,X\,10^5\,\hat{k}]$; $t = 0\,s$; $\boldsymbol{a} = ?$; $\boldsymbol{T} = ?$

(a) Mass of proton ; $m_p = 1.67\,X\,10^{-27}\,kg$

Charge of proton ; $e = 1.6\,X\,10^{-19}\,C$

From Newton's II law, acceleration is

No	Log
1.6	0.2041
1.67	0.2227
(-)	$\bar{1}$.9814
ALog	$9.581\,X\,10^{-1}$

$$\vec{a} = \frac{\vec{F}_B}{m_p} = \frac{q\,(\vec{v}\,X\,\vec{B})}{m_p}$$

$$\vec{a} = \frac{1.6\,X\,10^{-19}\,[(1.95\,X\,10^5\,\hat{i} + 2.00\,X\,10^5\,\hat{k})\,X\,0.500\,\hat{i}\,]}{1.67\,X\,10^{-27}}$$

$$\vec{a} = \frac{1.6\,X\,10^{-19}\,[1\,X\,10^5\,\hat{j}]}{1.67\,X\,10^{-27}} = \frac{1.6\,X\,10^{-14}}{1.67\,X\,10^{-27}}\,\hat{j}$$

$$\vec{a} = \frac{1.6\,X\,10^{13}}{1.67}\,\hat{j} = 9.581\,X\,10^{-1}\,X\,10^{13}\,\hat{j}$$

$$\boldsymbol{\vec{a} = 9.581\,X\,10^{12}\,\hat{j}} \quad (or) \quad \boldsymbol{a = 9.581\,X\,10^{12}\,m\,s^{-2}}$$

(b) Here this acceleration directed perpendicular to the magnetic field, due to the Lorentz force, the velocity component $v_z = 2.00\,X\,10^5\,\hat{k}$ along Z-axis alone continuously changed. Thus the path of proton is helical. The radius of helical path is

$$r = \frac{m_p\,v_z}{B\,e} = \frac{1.67\,X\,10^{-27}\,X\,2.00\,X\,10^5}{0.500\,X\,1.6\,X\,10^{-19}}$$

$$r = \frac{3.34\,X\,10^{-3}}{0.8} = \frac{33.4\,X\,10^{-3}}{8}$$

$$\boldsymbol{r = 4.175\,X\,10^{-3}m = 4.175\,mm}$$

Time period ;

$$T = \frac{2\,\pi}{\omega} = \frac{2\,\pi\,m_p}{B\,e}$$

$$T = \frac{2\ X\ 3.14\ X\ 1.67\ X\ 10^{-27}}{0.500\ X\ 1.6\ X\ 10^{-19}} = \frac{10.4876\ X\ 10^{-8}}{0.8} = \frac{104.876\ X\ 10^{-8}}{8}$$

$$\boldsymbol{T = 13.1095\ X\ 10^{-8}\ s\ = 13.11\ X\ 10^{-8}\ s}$$

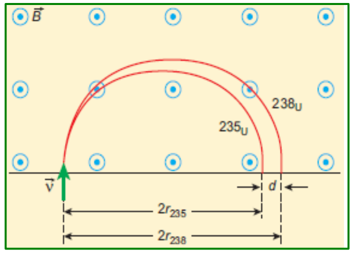
Hence, pitch of the helix is

$$X = v_x\,T = 1.95\ X\ 10^5\ X\ 13.11\ X\ 10^{-8} = 25.5645\ X\ 10^{-3}$$

$$\boldsymbol{X = 25.56\ X\ 10^{-3}\ m = 25.56\ mm}$$

The proton experiences appreciable acceleration in the magnetic field, hence the ***pitch of the helix is almost six times greater than the radius of the helix.***

21. Two singly ionized isotopes of uranium $^{235}_{92}U$ and $^{238}_{92}U$ (isotopes have same atomic number but different mass number) are sent with velocity $1.00 \times 10^5\ m\ s^{-1}$ into a magnetic field of strength 0.500 T normally. Compute the distance between the two isotopes after they complete a semi-circle. Also compute the time taken by each isotope to complete one semi-circular path.



(Given: masses of the isotopes: $m_{235} = 3.90 \times 10^{-25}$ kg and $m_{238} = 3.95 \times 10^{-25}$ kg)

Solution : $B = 0.500\ T$; $v = 1.00\ X\ 10^5\ m\ s^{-1}$; $m_{235} = 3.90\ X\ 10^{-25}\ kg$; $m_{238} = 3.95\ X\ 10^{-25}\ kg$; $q = |e| = 1.6\ X\ 10^{-19}\ C$; $d = ?$

♣ Since isotopes are singly ionized, they have equal charge which is equal to the charge of an electron

♣ The radius of the path of $^{235}_{92}U$ is r_{235}

$$r_{235} = \frac{m_{235}\ v}{B\ e} = \frac{3.90\ X\ 10^{-25}\ X\ 1.00\ X\ 10^5}{0.500\ X\ 1.6\ X\ 10^{-19}} = \frac{3.90\ X\ 10^{-1}}{0.8} = \frac{3.90}{8}$$

$$r_{235} = 0.4875\ m$$

Hence the diameter ; $d_{235} = 2\,r_{235} = 2\ X\ 0.4875 = 0.975\ m = 97.5\ cm$

♣ The radius of the path of $^{238}_{92}U$ is r_{235}

$$r_{238} = \frac{m_{238}\ v}{B\ e} = \frac{3.95\ X\ 10^{-25}\ X\ 1.00\ X\ 10^5}{0.500\ X\ 1.6\ X\ 10^{-19}} = \frac{3.95\ X\ 10^{-1}}{0.8} = \frac{3.95}{8}$$

$$r_{238} = 0.49375\ m$$

Hence the diameter ; $d_{238} = 2\,r_{238} = 2\ X\ 0.49375 = 0.9875\ m = 98.75\ cm$

♣ Therefore the separation distance between the isotopes is ;

$$\Delta \boldsymbol{d} = d_{238} - d_{235} = 0.9875 = 0.975 = \boldsymbol{0.0125\ m = 1.25\ cm}$$

♣ The time taken by each isotope to complete one semi-circular path are

$$t_{235} = \frac{d_{235}}{v} = \frac{0.975}{1.00\ X\ 10^5} = 0.975\ X\ 10^{-5} = 9.75\ X\ 10^{-6} s = 9.75\ \mu s$$

$$t_{238} = \frac{d_{238}}{v} = \frac{0.9875}{1.00\ X\ 10^5} = 0.9875\ X\ 10^{-5} = 9.875\ X\ 10^{-6} s = 9.875\ \mu s$$

22. Let E be the electric field of magnitude $6.0 \times 10^6\ N\ C^{-1}$ and B be the magnetic field magnitude 0.83 T. Suppose an electron is accelerated with a potential of 200 V, will it show zero deflection?. If not, at what potential will it show zero deflection?

Solution : $E = 6.0\ X\ 10^6\ N\ C^{-1}$; $B = 0.83\ T$; $V = 200\ V$; $v = ?$; $v_{200} = ?$; $V_0 = ?$

♣ At zero deflection, the velocity of electron,

$$v = \frac{E}{B} = \frac{6.0\ X\ 10^6}{0.83} = \boldsymbol{7.229\ X\ 10^6\ m\ s^{-1}}$$

No	Log
6.0	0.7782
0.83	$\bar{1}$.9191
(-) ALog	0.8591 7.229 X 10^0

♣ Since the accelerating potential is 200 V, the electron acquires kinetic energy because of this accelerating potential. Hence

$$\frac{1}{2}\ m\ v_{200}^2 = e\ V$$

$$v_{200}^2 = \frac{2\ e\ V}{m}$$

$$\therefore \quad v_{200} = \sqrt{\frac{2\ e\ V}{m}}$$

$$v_{200} = \sqrt{\frac{2\ X\ 1.6\ X\ 10^{-19}\ X\ 200}{9.1\ X\ 10^{-31}}} = \sqrt{\frac{640\ X\ 10^{12}}{9.1}}$$

$$\boldsymbol{v_{200} = 8.386\ X\ 10^6\ m\ s^{-1}}$$

No	Log
640	2.8062
9.1	0.9590
(-)	1.8472 X $\frac{1}{2}$
ALog	0.9236 8.386 X 10^0

♣ Since the speed $\boldsymbol{v_{200} > v}$, the electron is deflected towards direction of Lorentz force. So, in order to have null deflection, the potential V_0, we have to supply is

$$\frac{1}{2}\ m\ v^2 = e\ V_0$$

$$\therefore \quad V_0 = \frac{m\ v^2}{2\ e} = \frac{9.1\ X\ 10^{-31}\ X\ (7.229\ X\ 10^6)^2}{2\ X\ 1.6\ X\ 10^{-19}}$$

$$V_0 = \frac{9.1\ X\ 7.229\ X\ 7.229\ X\ 10^0}{3.2}$$

$$\boldsymbol{V_0 = 148.6\ V}$$

No	Log
9.1 7.229 7.229	0.9590 0.8591 0.8591
(+) 3.2	2.6772 0.5051
(-) ALog	2.1721 1.486 X 10^2

23. Suppose a cyclotron is operated to accelerate protons with a magnetic field of strength 1 T. Calculate the frequency in which the electric field between two Dees could be reversed.

Solution : B = 1 T ; $m_p = 1.67\ X\ 10^{-27}\ kg$; $q = |e| = 1.6\ X\ 10^{-19}\ C$; $f = ?$

♣ The frequency is,

$$f = \frac{B\ q}{2\ \pi\ m_p}$$

$$f = \frac{1\ X\ 1.6\ X\ 10^{-19}}{2\ X\ 3.14\ X\ 1.67\ X\ 10^{-27}}$$

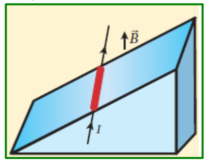
$$f = \frac{1.6\ X\ 10^8}{10.4876}$$

$$f = 1.525\ X\ 10^{-1}\ X\ 10^8 = 1.525\ X\ 10^7\ Hz$$

$$\boldsymbol{f = 15.25\ X\ 10^6\ Hz = 15.25\ MHz}$$

No	Log
1.6	0.2041
10.49	1.0208
(-) ALog	$\bar{1}$.1833 1.525 X 10^{-1}

24. A metallic rod of linear density is 0.25 kg m⁻¹ is lying horizontally on a smooth inclined plane which makes an angle of 45° with the horizontal. The rod is not allowed to slide down by flowing a current through it when a magnetic field of strength 0.25 T is acting on it in the vertical direction. Calculate the electric current flowing in the rod to keep it stationary.



Solution : $\frac{m}{l} = 0.25\ kg\ m^{-1}$; $B = 0.25\ T$; $I = ?$

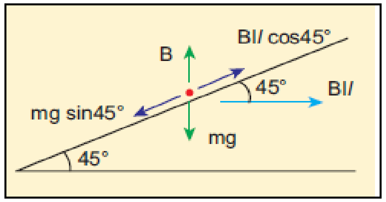
♣ Magnetic Lorentz force experienced by the current carrying conductor placed in magnetic field ; $F_B = B\,I\,l \sin 90° = B\,I\,l$

♣ The direction of this force $B\,I\,l$ is given by Fleming's left hand rule.

♣ This force is resolved in to two perpendicular components

1) $B\,I\,l\ cos\ 45°$ – along the inclined plane upwards

2) $B\,I\,l\ sin\ 45°$ – perpendicular to the incline plane downwards



♣ Similarly, weight mg also resolved in to two perpendicular components.

1) $mg\ cos\ 45°$ – perpendicular to the incline plane downwards

2) $mg\ sin\ 45°$ – along the inclined plane downwards

♣ For equilibrium of the rod ; $mg \sin 45° = B\,I\,l \cos 45°$

$$\therefore \quad I = \frac{mg \sin 45°}{B\,l \cos 45°} = \frac{\left(\frac{m}{l}\right) g}{B} = \frac{0.25\ X\ 9.8}{0.25} = \mathbf{9.8\ A}$$

♣ So, we need to supply current of 9.8 A to keep the metallic rod stationary.

25. The coil of a moving coil galvanometer has 5 turns and each turn has an effective area of 2 × 10⁻² m². It is suspended in a magnetic field whose strength is 4 × 10⁻² Wb m⁻². If the torsional constant K of the suspension fibre is 4 × 10⁻⁹ N m deg⁻¹.

(a) Find its current sensitivity in division per micro - ampere.

(b) Calculate the voltage sensitivity of the galvanometer for it to have full scale deflection of 50 divisions for 25 mV.

(c) Compute the resistance of the galvanometer.

Solution : $n = 5$; $A = 2\ X\ 10^{-2}\ m^2$; $B = 4\ X 10^{-2}\ Wb\ m^{-2}$; $K = 4\ X\ 10^{-9}\ N\ m\ deg^{-1}$

(a) Current sensitivity,

$$\boldsymbol{I_S} = \frac{\theta}{I} = \frac{N\,B\,A}{K} = \frac{5\ X\ 4\ X 10^{-2}\ X\ 2\ X\ 10^{-2}}{4\ X\ 10^{-9}} = \frac{1}{10^{-6}}\ deg/A = \mathbf{1\ deg/\mu A}$$

(b) Voltage sensitivity,

$$\boldsymbol{V_S} = \frac{\theta}{V} = \frac{50}{25\ X\ 10^{-3}} = 2\ X\ 10^3\ deg/volt$$

(c) Resistance of the galvanometer,

$$\boldsymbol{R_g} = \frac{\boldsymbol{I_S}}{\boldsymbol{V_S}} = \frac{\left(\frac{1}{10^{-6}}\right)}{2\ X\ 10^3} = \frac{1}{2\ X\ 10^3\ X\ 10^{-6}} = 0.5\ X\ 10^3 = \mathbf{500\ \Omega} = \mathbf{0.5\ k\Omega}$$

26. The resistance of a moving coil galvanometer is made twice its original value in order to increase current sensitivity by 50%. Find the percentage change in voltage sensitivity.

Solution :

♣ Let I_S be the initial current sensitivity. If current sensitivity is increased by 50%, then new current sensitivity,

$$\boldsymbol{I_S'} = I_S + 50\%\ I_S = I_S\left[1 + \frac{50}{100}\right] = I_S\left[1 + \frac{1}{2}\right] = \frac{3}{2}\boldsymbol{I_S} = 1.5\,I_S$$

♣ Let V_S be the initial Voltage sensitivity. When the resistance is doubled, then new voltage sensitivity is

$$\boldsymbol{V_S'} = \frac{I_S'}{R_g'} = \frac{\left(\frac{3}{2}\right) I_S}{2\,R_g} = \frac{3}{4}\boldsymbol{V_S} = 0.75\,V_S$$

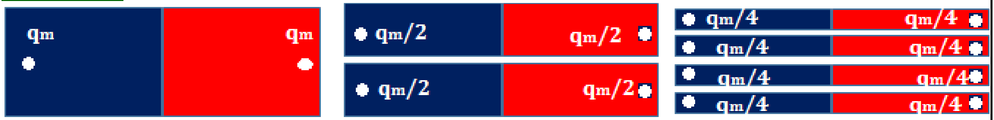
♣ Hence the voltage sensitivity decreases. The percentage decrease in voltage sensitivity is

$$\frac{\boldsymbol{V_S - V_S'}}{\boldsymbol{V_S}}\boldsymbol{X\ 100\ \%} = \frac{\boldsymbol{V_S - 0.75\,V_S}}{\boldsymbol{V_S}}\ \boldsymbol{X\ 100\% = 0.25\ X\ 100\% = 25\ \%}$$

EXERCISE PROBLEMS

1. **A bar magnet having a magnetic moment $\vec{p}_m$ is cut into four pieces i.e., first cut into two pieces along the axis of the magnet and each piece is further cut along the axis into two pieces. Compute the magnetic moment of each piece.**

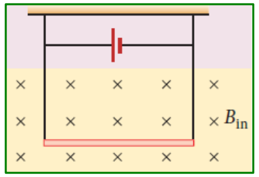
Solution :



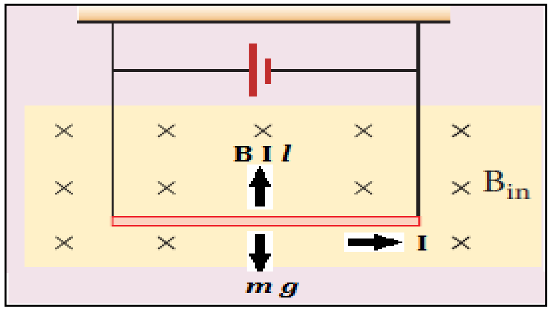
- ♣ Initial magnetic moment of the magnet ; $\vec{p}_m = q_m \vec{d} = q_m\, 2\vec{l}$
- ♣ When it cut along the axis into four pieces, the pole strength of each piece becomes $\frac{q_m}{4}$, but the magnetic length will not change. Hence magnetic moment of each piece,

$$\vec{p}_m{}' = \frac{q_m}{4}\vec{d} = \frac{1}{2}\left(q_m\, 2\vec{l}\right) = \frac{1}{4}\vec{p}_m$$

2. **A conductor of linear mass density 0.2 g m^{-1} suspended by two flexible wire as shown in figure. Suppose the tension in the supporting wires is zero when it is kept inside the magnetic field of 1 T whose direction is into the page. Compute the current inside the conductor and also the direction of the current. Assume g = 10 m s^{-2}**



Solution : $\frac{m}{l} = 0.2\ g\ m^{-1} = 0.2\ X\ 10^{-3}\ kg\ m^{-1}$; $B = 1\,T$; $g = 10\ m\ s^{-2}$



- ♣ Weight of the conductor due to gravity in downward direction,

$$F_g = m\,g$$

- ♣ Magnetic Lorentz force acting perpendicular to conductor in upward direction,

$$F_B = B\,I\,l$$

- ♣ When the tension of the supporting wire becomes zero, we have

$$F_B = F_g$$

$$B\,I\,l = m\,g$$

$$I = \frac{m\,g}{B\,l} = \frac{\left(\frac{m}{l}\right)g}{B} = \frac{0.2\ X\ 10^{-3}\ X\ 10}{1} = 2\ X\ 10^{-3}\ A = 2\ m\,A$$

3. **A circular coil with cross-sectional area 0.1 cm² is kept in a uniform magnetic field of strength 0.2 T. If the current passing in the coil is 3 A and plane of the loop is perpendicular to the direction of magnetic field. Calculate**
(a) total torque on the coil
(b) total force on the coil
(c) average force on each electron in the coil due to the magnetic field. (The free electron density for the material of the wire is 10²⁸ m⁻³).

Solution : $A = 0.1\ cm^2 = 0.1\ X\ 10^{-4}\ m^2$; $B = 0.2\,T$; $I = 3\,A$; $n = 10^{28}\ m^{-3}$; $\theta = 0°$

(a) Total torque on the coil

$$\tau = p_m\,B\sin\theta = I\,A\,B\sin\theta$$

$$\tau = 3\ X\ 0.1\ X\ 10^{-4}\ X\ 0.2\ X\sin 0° \qquad [\because \sin 0° = 0]$$

$$\boldsymbol{\tau = 0}$$

(b) Total force on the coil,

$$F = B\,I\,l\sin\theta$$

$$\boldsymbol{F = 0} \qquad [\because \sin 0° = 0]$$

(c) Charge of electron ; $q = |e| = 1.6\ X\ 10^{-19}\ C$

If l is the length of the coil, the drift velocity ; $v_d = \frac{l}{t}$

Lorentz force on each electron,

$$F_B = B\,q\,v = B\,e\,v_d \qquad [\because I = n\,A\,e\,v_d]$$

$$F_B = B\,e\,\frac{I}{n\,A\,e} = \frac{B\,I}{n\,A}$$

$$F_B = \frac{0.2\ X\ 3}{10^{28}\ X\ 0.1\ X\ 10^{-4}} = \frac{0.6}{0.1}\ X\ 10^{-24} = \boldsymbol{6\ X\ 10^{-24}\ N}$$

4. **A bar magnet is placed in a uniform magnetic field whose strength is 0.8 T.If the bar magnet is oriented at an angle 30° with the external field experiences a torque of 0.2 Nm. Calculate: (a) the magnetic moment of the magnet (b) the work done by the magnetic field in moving it from most stable configuration to the most unstable configuration and also compute the work done by the applied magnetic field in this case.**

Solution : $B = 0.8\,T$; $\theta = 30°$; $\tau = 0.2\ N\,m$

(a) We know that, the torque ; $\tau = p_m\,B\sin\theta$

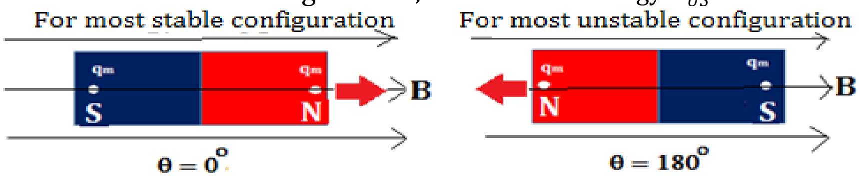
Hence magnetic moment is given by,

$$p_m = \frac{\tau}{B\sin\theta} = \frac{0.2}{0.8\ X\sin 30°} = \frac{0.2}{0.8\ X\ \frac{1}{2}} = \frac{0.2}{0.4} = \frac{2}{4}$$

$$\boldsymbol{p_m = 0.5\ A\ m^2}$$

(b) For most stable configuration ; $\theta = 0°$ and energy U_S

For most unstable configuration ; $\theta = 180$ and energy U_{US}



- From the figure,

$$U_S = -p_m B\cos\theta = -p_m B\cos 0° = -p_m B$$
$$U_{US} = -p_m B\cos\theta = -p_m B\cos 180° = -p_m B(-1) = p_m B$$

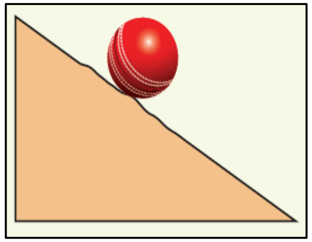
- Hence the work done by the magnetic field in moving it from most stable configuration to the most unstable configuration is,

$$W_F = U_{US} - U_S = p_m B - (-p_m B) = 2\,p_m B$$
$$W_F = 2\ X\ 0.5\ X\ 0.8$$
$$\boldsymbol{W_F = 0.8\ J}$$

- Work done by the applied magnetic field,

$$W_B = U_S - U_{US} = -p_m B - p_m B = -2\,p_m B$$
$$W_B = -2\ X\ 0.5\ X\ 0.8$$
$$W_B = \boldsymbol{-0.8\ J}$$

5. A non - conducting sphere has a mass of 100 g and radius 20 cm. A flat compact coil of wire with turns 5 is wrapped tightly around it with each turns concentric with the sphere. This sphere is placed on an inclined plane such that plane of coil is parallel to the inclined plane. A uniform magnetic field of 0.5 T exists in the region in vertically upward direction. Compute the current *I* required to rest the sphere in equilibrium.



Solution : $M = 100$ g = 0.1 kg ; R = 20 cm = $20\ X\ 10^{-2}$ m ; N = 5 ; B = 0.5 T ; I = ?

- Here sphere be at two types of equilibrium. They are,
  - (i) Straight line equilibrium
  - (ii) Rotational equilibrium
- Under straight line equilibrium ; $f_S = M\,g\sin\theta$ ------- (1)
- Torque on the sphere due to magnetic field about its centre ;

$\tau = p_m B\sin\theta$ (anticlockwise direction)

Torque on the sphere due to friction about its centre ;

$\tau = f_S R$ (clockwise direction)

Under rotational equilibrium ; $f_S R = p_m B\sin\theta$ ------- (2)

- Put equation (1) in (2)

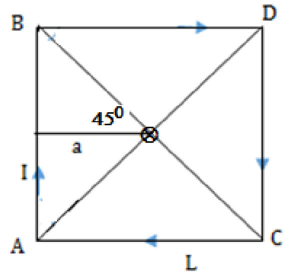
$$M\,g\sin\theta\,R = p_m B\sin\theta$$
$$M\,g\,R = p_m B \qquad [\because p_m = N\,I\,A]$$
$$M\,g\,R = N\,I\,A\,B$$
$$M\,g\,R = N\,I\,(\pi R^2)\,B$$
$$I = \frac{M\,g}{N\,\pi\,R\,B}$$
$$I = \frac{0.1\ X\ 10}{5\ X\ \pi\ X\ 20\ X\ 10^{-2}\ X\ 0.5}$$
$$I = \frac{10^2}{50\ X\ \pi} = \frac{100}{50\ X\ \pi}$$
$$\boldsymbol{I = \frac{2}{\pi}\ A}$$

6. Calculate the magnetic field at the centre of a square loop which carries a current of 1.5 A, length of each side being 50 cm.

Solution : $L = 50\ cm = 50\ X\ 10^{-2}\ m$; $I = 1.5\ A$ $a = \frac{L}{2} = 25cm = 25\ X\ 10^{-2}\ m$

$\phi_1 = \phi = 45°$; $\phi_2 = 180° - \phi$; $\theta = 45°$

- Let the square loop is made up of four straight conductors AB, BD, DC and CA
- From Biot - Savart law, the magnetic field at a distance 'a' due to straigtht current carrying conductor AB is,



$$B_{AB} = \frac{\mu_o\,I}{4\,\pi\,a}[\sin\phi_1 + \sin\phi_2]$$
$$B_{AB} = \frac{4\,\pi\ X\ 10^{-7}\ X\ 1.5}{4\,\pi\ X\ 25\ X\ 10^{-2}}[\sin 45° + \sin 45°]$$
$$B_{AB} = \frac{10^{-5}\ X\ 1.5}{25}[2\ \sin 45°]$$
$$B_{AB} = 0.06\ X\ 10^{-5}\ X\ 2\ X\ \frac{1}{\sqrt{2}} = 6\sqrt{2}\ X\ 10^{-7}\ T$$

- Similarly, magnetic field due to BD, DC and CA

$$B_{BD} = 6\sqrt{2}\ X\ 10^{-7}\ T$$
$$B_{DC} = 6\sqrt{2}\ X\ 10^{-7}\ T$$
$$B_{CA} = 6\sqrt{2}\ X\ 10^{-7}\ T$$

- From Fleming's left hand rule, the magnetic field at the centre, due to all four conductors is directed perpendicularly inwards to the plane of the paper. Hence the total magnetic field

$$B = B_{AB} + B_{BD} + B_{DC} + B_{CA}$$
$$B = 4\ X\ 6\sqrt{2}\ X\ 10^{-7}$$
$$B = 24\ X\ 1.414\ X\ 10^{-7}$$
$$B = 33.936\ X\ 10^{-7} = 3.3936\ X\ 10^{-6}\ T$$
$$\boldsymbol{B = 3.4\ X\ 10^{-6}\ T}$$