# 12th IMPORTANT 5 MARKS

14) Solve the following systems of linear equations by Cramer's rule:
$\frac{3}{x}-\frac{4}{y}-\frac{2}{z}-0, \frac{1}{x}+\frac{2}{y}+\frac{1}{z}-2=0, \frac{2}{x}-\frac{5}{y}-\frac{4}{z}+1=0$

15)
If z=x+iy is a complex number such that Im $\left(\frac{2z+1}{iz+1}\right)=0$ show that the locus of z is $2x^2+2y^2+x-2y=0$

16)
If z=x+iy and arg $\left(\frac{z-i}{z+2}\right)=\frac{\pi}{4}$, then show that $x^2+y^2+3x-3y+2=0$

17)
If $2\cos\alpha = x+\frac{1}{x}$ and $2\cos\beta = y+\frac{1}{y}$, show that

i) $\frac{x}{y}+\frac{y}{x}=2\cos(\alpha-\beta)$.

ii) $xy-\frac{1}{xy}=2i\sin(\alpha+\beta)$

iii) $\frac{x^m}{y^n}-\frac{y^n}{x^m}=2i\sin(m\alpha-n\beta)$

iv) $x^my^n+\frac{1}{x^my^n}=2\cos(m\alpha+n\beta)$

18)
If z=x+iy and arg $\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2}$,then show that $x^2+y^2=1$.

19) If 2+i and $3-\sqrt{2}$ are roots of the equation $x^6-13x^5+62x^4-126x^3+65x^2+127x-140=0$, find all roots.

20) Solve the equation $(x-2)(x-7)(x-3)(x+2)+19=0$

21) Solve the equation $(2x-)(6x-1)(3x-2)(x-12)-7=0$

22) Determine k and solve the equation $2x^3-6x^2+3x+k=0$ if one of its roots is twice the sum of the other two roots.

23) Find all zeros of the polynomial $x^6-3x^5-5x^4+22x^3-39x^2-39x+135$, if it is known that 1+2i and $\sqrt{3}$ are two of its zeros.

24) Solve the following equation: $x^4-10x^3+26x^2-10x+1=0$

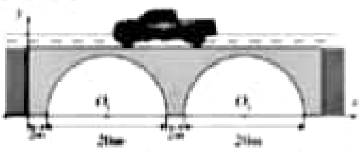
25)
Solve $\tan^{-1}\left(\frac{x-1}{x-2}\right)+\tan^{-1}\left(\frac{x+1}{x+2}\right)=\frac{\pi}{4}$

26)
Solve $\cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right)=\sin\left\{\cot^{-1}\left(\frac{3}{4}\right)\right\}$

27)
Provethat $\tan^{-1}\left(\frac{1-x}{1+x}\right)-\tan^{-1}\left(\frac{1-y}{1+y}\right)=\sin^{-1}\left(\frac{y-x}{\sqrt{1+x^2}.\sqrt{1+y^2}}\right)$

28) Find the equation of the circle passing through the points(1,1), (2,-1) , and(3,2) .

29) A road bridge over an irrigation canal have two semi circular vents each with a span of 20m and the supporting pillars of width 2m. Use Fig.5.16 to write the equations that model the arches.



30) Find the foci, vertices and length of major and minor axis of the conic $4x^2+36y^2+40x-288y+532=0$.

$$\left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 36 & 6 & 1 & 133 \\ 81 & 9 & 1 & 208 \end{array}\right] \xrightarrow{R_2 \to R_2 - 4R_1, R_3 \to R_3 - 9R_1} \left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 0 & -6 & -3 & -123 \\ 0 & -18 & -8 & -368 \end{array}\right] \xrightarrow{R_2 \to R_2 \div (-3), R_3 \div (-2)} \left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 9 & 4 & 184 \end{array}\right]$$

$$\xrightarrow{R_3 \to 2R_3} \left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 18 & 8 & 368 \end{array}\right] \xrightarrow{R_3 \to R_3 - 9R_2} \left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 0 & -1 & -1 \end{array}\right] \xrightarrow{R_3 \to (-1)R_3} \left[\begin{array}{ccc|c} 9 & 3 & 1 & 64 \\ 0 & 2 & 1 & 41 \\ 0 & 0 & 1 & 1 \end{array}\right].$$

Writing the equivalent equations from the row-echelon matrix, we get

$9a + 3b + c = 64$, $2b + c = 41$, $c = 1$.

By back substitution, we get $c = 1$, $b = \frac{(41-c)}{2} = \frac{(41-1)}{2} = 20$, $a = \frac{64-3b-c}{9} = \frac{64-60-1}{9} = \frac{1}{3}$.

So, we get $v(t) = \frac{1}{3}t_2 + 20t + 1$. Hence, $v(15) = \frac{1}{3}(225) + 20(15) + 1 = 75 + 300 + 1 = 376$.

6) Let $P(x) = ax^2+bx+c$

Given $P(-3) = 21$

[$\because$ $P(x) \div x+3$, the remainder is 21]

$\rightarrow a(-3)2+b(-3)+c=21$

$\rightarrow 9a-3b+c=21$

Also, $P(5)=61$

$\rightarrow a(5)2+b(5)+c=61$

[using remainder theorem]

$\rightarrow 25x+5b+c=61$ ..........(2)

and $P(1)=9$

$\rightarrow a(1)2+b(1)+c=9$

$\rightarrow a+b+c=9$ .........(3)

Reducing the augment matrix to an equivalent row-echelon form using elementary row operations, we get

$$\left[\begin{array}{ccc|c} 9 & - & 1 & 21 \\ 25 & 5 & 1 & 61 \\ -1 & 1 & 1 & 9 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 25 & 5 & 1 & 61 \\ 9 & -3 & 1 & 21 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 9R_1 R_2 \to R_2 - 25R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -20 & -24 & -164 \\ 0 & -12 & -8 & -60 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 \div R_3 \to R_3 \div 4} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & -6 & -41 \\ 0 & -3 & -2 & -15 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - \frac{3}{5}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & -6 & -41 \\ 0 & 0 & \frac{8}{5} & \frac{48}{5} \end{array}\right]$$

$$\xrightarrow{R_3 \to 5R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 0 & -5 & -6 & -41 \\ 0 & 0 & 8 & 48 \end{array}\right]$$

Writing the equivalent equations from the row-echelon matrix we get,

$a+b+c = 9$ ....(1)

$-5b-6c = -41$ .....(2)

$8c=48$

$\rightarrow c=\frac{48}{8}=6$

Substituting $c=6$ in (2) we get,

adj A =
$$\begin{bmatrix} (21-16) & -(-18+8) & (24-14) \\ -(-18+8) & (24-4) & -(32+12) \\ (24-14) & -(-32+12) & (56-36) \end{bmatrix}^T = \begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$$

So, we get

A(adj A) =
$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}\begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 40-60+20 & 80-120+40 & 80-120+40 \\ -30+70-40 & -60+140-80 & -60+140-80 \\ 10-40+30 & 20-80+60 & 20-80+60 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0I_3 = |A|I_3 \,,$$

Similarly, we get

(adj A)A =
$$\begin{bmatrix} 5 & 10 & 10 \\ 10 & 20 & 20 \\ 10 & 20 & 20 \end{bmatrix}\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 40-60+20 & -30+70-40 & 10-40+30 \\ 80-120+40 & -60+140-80 & 20-80+60 \\ 80-120+40 & -60+140-80 & 20-80+60 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = 0I_3 = |A|I_3 \,.$$

Hence, A(adj A) = (adj A)A = $|A|I_3$.

2) Applying Gauss-Jordan method, we get

$$[A \mid I_3] = \left[\begin{array}{ccc|ccc} 2 & 1 & 1 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_1 \to \frac{1}{2}R_1} \left[\begin{array}{ccc|ccc} 1 & (1/2) & (1/2) & (1/2) & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 - 2R_1]{R_2 \to R_2 - 3R_1} \left[\begin{array}{ccc|ccc} 1 & (1/2) & (1/2) & (1/2) & 0 & 0 \\ 0 & (1/2) & -(1/2) & -(3/2) & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \to 2R_2} \left[\begin{array}{ccc|ccc} 1 & (1/2) & (1/2) & (1/2) & 0 & 0 \\ 0 & 1 & -1 & -3 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - \frac{1}{2}R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 2 & -1 & 0 \\ 0 & 1 & -1 & -3 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right] \xrightarrow[R_2 \to R_2 + R_3]{R_1 \to R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & -1 \\ 0 & 1 & 0 & -4 & 2 & 1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{array}\right]$$

SO, $A^{-1} = \begin{bmatrix} 3 & -1 & -1 \\ -4 & 2 & 1 \\ -1 & 0 & 1 \end{bmatrix}$.

3) We find AB =
$$\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -4+4+8 & 4-8+4 & -4-8+12 \\ -7+1+6 & 7-2+3 & -7-2+9 \\ 5-3-2 & -5+6-1 & 5+6-3 \end{bmatrix} =$$

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I_3$$

and BA =
$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix} = \begin{bmatrix} -4+7+5 & 4-1-3 & 4-3-1 \\ -4+14-10 & 4-2+6 & 4-6+2 \\ -8-7+15 & 8+1-9 & 8+3-3 \end{bmatrix} =$$

# 12th IMPORTANT 5 MARKS

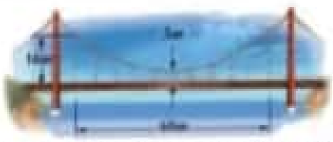
31) Find the centre, foci, and eccentricity of the hyperbola $11x^2-25y^2-44x+50y-256=0$

32) Show that the line $x-y+4=0$ is a tangent to the ellipse $x^2+3y^2=12$. Also find the coordinates of the point of contact.

33) Two coast guard stations are located 600 km apart at points A(0,0) and B(0,600). A distress signal from a ship at P is received at slightly different times by two stations. It is determined that the ship is 200 km farther from station A than it is from station B. Determine the equation of hyperbola that passes through the location of the ship.

34) At a water fountain, water attains a maximum height of 4m at horizontal distance of 0 5 . m from its origin. If the path of water is a parabola, find the height of water at a horizontal distance of 0.75m from the point of origin.

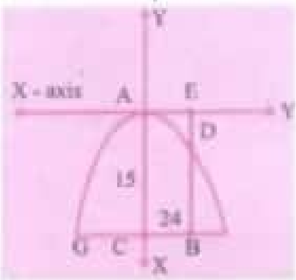
35) Parabolic cable of a 60m portion of the roadbed of a suspension bridge are positioned as shown below. Vertical Cables are to be spaced every 6m along this portion of the roadbed. Calculate the lengths of first two of these vertical cables from the vertex.



36) Assume that water issuing from the end of a horizontal pipe, 7 5 . m above the ground, describes a parabolic path. The vertex of the parabolic path is at the end of the pipe. At a position 2 5 . m below the line of the pipe, the flow of water has curved outward 3m beyond the vertical line through the end of the pipe. How far beyond this vertical line will the water strike the ground?

37) On lighting a rocket cracker it gets projected in a parabolic path and reaches a maximum height of 4m when it is 6m away from the point of projection. Finally it reaches the ground 12m away from the starting point. Find the angle of projection.

38) The guides of a railway bridge is a parabola with its vertex at the highest point 15 m above the ends. If the span is 120 m, find the height of the bridge at 24 m from the middle point.




39) By vector method, prove that $\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$

40) With usual notations, in any triangle ABC, prove by vector method that $\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$

41) Prove by vector method that $\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$

42) Prove by vector method that the perpendiculars (attitudes) from the vertices to the opposite sides of a triangle are concurrent.

43) Using vector method, prove that $\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$

44) Prove by vector method that $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

45) If $\vec{a}=\hat{i}-\hat{j}, \vec{b}=\hat{i}-\hat{j}-4\hat{k}, \vec{c}=3\hat{j}-\hat{k}$ and $\vec{d}=2\hat{i}+5\hat{j}+\hat{k}$

(i) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{b},\vec{d}]\vec{c}-[\vec{a},\vec{b},\vec{c}]\vec{d}$

(ii) $(\vec{a}\times\vec{b})\times(\vec{c}\times\vec{d})=[\vec{a},\vec{c},\vec{d}]\vec{b}-[\vec{b},\vec{c},\vec{d}]\vec{a}$

46) Find the non-parametric form of vector equation, and Cartesian equation of the plane passing through the point (2,3,6) and parallel to the straight lines $\frac{x-1}{2}=\frac{y+1}{3}=\frac{z-3}{1}$ and $\frac{x+3}{2}=\frac{y-3}{-5}=\frac{z+1}{-3}$

47) Find the parametric form of vector equation, and Cartesian equations of the plane passing through the points (2, 2,1), (9,3,6) and perpendicular to the plane $2x+6y+6z=9$

48) Find the non-parametric form of vector equation of the plane passing through the point (1, -2, 4) and perpendicular to the plane $x+2y-3z=11$ and parallel to the line $\frac{x+7}{3}=\frac{y+3}{-1}=\frac{z}{1}$

**IMPORTANT 5 marks**

12th Standard

Maths

100 x 5 = 500

1) If A = $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{bmatrix}$, verify that $A(\text{adj } A)=(\text{adj } A)A = |A|\, I_3$.

2) Find the inverse of A = $\begin{bmatrix} 2 & 1 & 1 \\ 3 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$ by Gauss-Jordan method.

3) If A = $\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}$ and B = $\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}$, find the products AB and BA and hence solve the system of equations $x - y + z = 4, x - 2y - 2z = 9, 2x + y + 3z = 1$.

4) In a T20 match, Chennai Super Kings needed just 6 runs to win with 1 ball left to go in the last over. The last ball was bowled and the batsman at the crease hit it high up. The ball traversed along a path in a vertical plane and the equation of the path is $y = ax^2 + bx + c$ with respect to a xy-coordinate system in the vertical plane and the ball traversed through the points (10, 8), (20, 16) (30, 18) can you conclude that Chennai Super Kings won the match?
Justify your answer. (All distances are measured in metres and the meeting point of the plane of the path with the farthest boundary line is (70, 0).)

5) The upward speed v(t)of a rocket at time t is approximated by $v(t) = at^2 + bt + c \le t \le 100$ where a, b and c are constants. It has been found that the speed at times t = 3, t = 6, and t = 9 seconds are respectively, 64, 133, and 208 miles per second respectively. Find the speed at time t = 15 seconds. (Use Gaussian elimination method.)

6) If $ax^2 + bx + c$ is divided by $x + 3$, $x - 5$, and $x - 1$, the remainders are 21,61 and 9 respectively. Find a,b and c. (Use Gaussian elimination method.)

7) Investigate for what values of λ and μ the system of linear equations
$x + 2y + z = 7, x + y + \lambda z = \mu, x + 3y - 5z = 5$ has
(i) no solution
(ii) a unique solution
(iii) an infinite number of solutions

8) Find the value of k for which the equations $kx - 2y + z = 1, x - 2ky + z = -2, x - 2y + kz = 1$ have
(i) no solution
(ii) unique solution
(iii) infinitely many solution

9) Investigate the values of λ and m the system of linear equations $2x + 3y + 5z = 9, 7x + 3y - 5z = 8, 2x + 3y + \lambda z = \mu$, have
(i) no solution
(ii) a unique solution
(iii) an infinite number of solutions.

10) Determine the values of λ for which the following system of equations $(3\lambda - 8)x + 3y + 3z = 0, 3x + (3\lambda - 8)y + 3z = 0, 3x + 3y + (3\lambda - 8)z = 0$. has a non-trivial solution.

11) By using Gaussian elimination method, balance the chemical reaction equation: $C_5H_8 + O_2 \rightarrow CO_2 + H_2O$. (The above is the reaction that is taking place in the burning of organic compound called isoprene.)

12) If the system of equations $px + by + cz = 0, ax + qy + cz = 0, ax + by + rz = 0$ has a non-trivial solution and $p \ne a, q \ne b, r \ne c$, prove that $\frac{p}{p-a} + \frac{q}{q-b} + \frac{r}{r-c} = 2$.

13) By using Gaussian elimination method, balance the chemical reaction equation:
$C_2H_5 + O_2 \rightarrow H_2O + CO_2$

# 12th IMPORTANT 5 MARKS

49) Find the parametric vector, non-parametric vector and Cartesian form of the equations of the plane passing through the points $(3,6,-2)$, $(-1,-2,6)$, and $(6,-4,-2)$.

50) Salt is poured from a conveyer belt at a rate of 30 cubic metre per minute forming a conical pile with a circular base whose height and diameter of base are always equal. How fast is the height of the pile increasing when the pile is 10 metre high?

51) A road running north to south crosses a road going east to west at the point P. Car A is driving north along the first road, and car B is driving east along the second road. At a particular time car A 10 kilometres to the north of P and traveling at 80 km/hr, while car B is 15 kilometres to the easst of P and traveling at 100 km/hr. How fast is the distance between the two cars changing?

52) A conical water tank with vertex down of 12 metres height has a radius of 5 metres at the top. If water flows into the tank at a rate 10 cubic m/min, how fast is the depth of the water increases when the water is 8 metres deep?

53) A police jeep, approaching an orthogonal intersection from the northern direction, is chasing a speeding car that has turned and moving straight east. When the jeep is 0.6 km north of the intersection and the car is 0.8 km to the east. The police determine with a radar that the distance between them and the car is increasing at 20 km/hr. If the jeep is moving at 60 km/hr at the instant of measurement, what is the speed of the car?

54) Find the acute angle between $y = x^2$ and $y = (x-3)^2$.

55) A rectangular page is to contain 24 $cm^2$ of print. The margins at the top and bettemof the page are 1.5 cm and the margins at other sides of the page is 1 cm. What should be the dimensions of the page so that the area of the paper used is minimum.

56) Expand tan x in ascending powers of x upto $5^{th}$ power for $\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$

57) Write the Maclaurin series expansion of the following function
$\tan^{-1}(x); -1 \le x \le 1$

58) Evaluate the following limit, if necessary use l'Hôpital Rule
$lim_{x\to 0^+} (\cos x)^{\frac{1}{x^2}}$

59) For the function $f(x)=4x^3+3x^2-6x+1$ find the intervals of monotonicity, local extrema, intervals of concavity and points of inflection.

60) Consider $g(x,y) = \frac{2x^2y}{x^2+y^2}$, if $(x,y) \neq (0,0)$ and $g(0,0) = 0$ Show that g is continuous on $R^2$

61) For each of the following functions find the $f_x$, $f_y$, and show that $f_{xy} = f_{yx}$
$f(x,y) = \frac{3x}{y+\sin x}$

62) If $U(x, y, z) = \log(x^3 + y^3 + z^3)$, find $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y} + \frac{\partial U}{\partial z}$

63) Let $w(x, y, z) = \frac{1}{\sqrt{x^2+y^2+z^2}}$, $(x, y, z) \neq (0, 0, 0)$. Show that $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = 0$

64) If $(x,y) = \log\left(\frac{x^2+y^2}{x+y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

65) If $w(x,y,z) = \log\left(\frac{5x^3y^4+7y^2xz^4-75y^3z^4}{x^2+y^2}\right)$ find $x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} + z\frac{\partial w}{\partial z}$

66) If $u=\sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, Show that x $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$

67) If $u = \tan^{-1}\left(\frac{x^3+y^3}{x-y}\right)$

Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ sin 2u.

68) Evaluate: $\int_0^{\frac{\pi}{2}} (\sqrt{\tan x} + \sqrt{\cot x})dx$

69) Prove that $\int_0^{\frac{\pi}{2}} \frac{\sin 2x dx}{\sin^4 x+\cos^4 x} = \frac{\pi}{4}$

# 12th IMPORTANT 5 MARKS

(i) What was the temperature of the coffee at 10.15A.M.?

(ii) The woman likes to drink coffee when its temperature is between $130^{0}$F and $140^{0}$F.between what times should she have drunk the coffee?

91) Suppose the amount of milk sold daily at a milk booth is distributed with a minimum of 200 litres and a maximum of 600 litres with probability density function

$$\begin{cases} k & 200 \le x \le 600 \\ 0 & otherwise \end{cases}$$

Find

(i) the value of k

(ii) the distribution function

(iii) the probability that daily sales will fall between 300 litres and 500 litres?

92) The probability density function of X is given

$$f(x) = \begin{cases} Ke^{\frac{-x}{3}} & for \quad x > 0 \\ 0 & for \quad x \le 0 \end{cases}$$

Find

(i) the value of k

(ii) the distribution function.

(iii) $P(X < 3)$

(iv) $P(5 \le X)$

(v) $P(X \le 4)$

93) A retailer purchases a certain kind of' electronic device from a manufacturer. The manufacturer ,indicates that the defective rate of the device is 5%. The inspector of the retailer randomly picks 10 items from a shipment. What is the probability that there will be (i) at least one defective item (ii) exactly two defective items.

94) Suppose that f (x) given below represents a probability mass function

| | | | | | | |
|---|---|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| f(x) | $c^2$ | $2c^2$ | $3c^2$ | $4c^2$ | c | 2c |

Find

(i) the value of c

(ii) Mean and variance.

95) On the average, 20% of the products manufactured by ABC Company are found to be defective. If we select 6 of these products at random and X denote the number of defective products find the probability that (i) two products are defective (ii) at most one product is defective (iii) at least two products are defective.

96) The mean and variance of a binomial distribution are 4 and 2 respectively. Find the probability of at least 6 success.

97) Let M=$\left\{\begin{pmatrix} x & x \\ x & x \end{pmatrix} : x \in R - \{0\}\right\}$ and let * be the matrix multiplication. Determine whether M is closed under ∗. If so, examine the commutative and associative properties satisfied by ∗ on M.

98) Let A be Q\{1}. Define ∗ on A by x*y = x + y − xy . Is ∗ binary on A? If so, examine the commutative and associative properties satisfied by ∗ on A.

99) Using truth table check whether the statements $\neg(p \vee q) \vee (\neg p \wedge q)$ and $\neg p$ are logically equivalent.

100) Let A be Q\{1}. Define ∗ on A by x*y = x + y − xy . Is ∗ binary on A? If so, examine the existence of identity, existence of inverse properties for the operation ∗ on A.

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$100 \times 5 = 500$

1) We find that $|A| = \begin{vmatrix} 8 & -6 & 2 \\ -6 & 7 & 4 \\ 2 & -4 & 3 \end{vmatrix} = 8(21 - 16) + 6(-18 + 8) + 2(24 - 14) = 40 - 60 + 20 = 0$

By the definition of adjoint, we get

# 12th IMPORTANT 5 MARKS

$$\begin{bmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{bmatrix} = 8I_3$$

So we get AB = BA = $8I_3$. That is, $(\frac{1}{8}A)B = B(\frac{1}{8}A) = I_3$. Hence, $B^{-1} = \frac{1}{8}A$.

Writing the given system of equations in matrix form, we get

$$\begin{bmatrix} 1 & -1 & 1 \\ 1 & -2 & -2 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}. \text{ That is } B\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix}.$$

So,

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^{-1}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} =$$

$$\left(\frac{1}{8}A\right)\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -4 & 4 & 4 \\ -7 & 1 & 3 \\ 5 & -3 & -1 \end{bmatrix}\begin{bmatrix} 4 \\ 9 \\ 1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} -16+36+4 \\ -28+9+3 \\ 20-27-1 \end{bmatrix} = \frac{1}{8}\begin{bmatrix} 24 \\ -16 \\ -8 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$

Hence, the solution is ($x = 3$, $y = -2$, $z = -1$).

4) The path $y = ax^2 + bx + c$ passes through the points (10, 8), (20, 16), (40, 22). So, we get the system of equations $100a + 10b + c = 8$, $400a + 20b + c = 16$, $1600a + 40b + c = 22$. To apply Cramer's rule, we find

$$\Delta = \begin{vmatrix} 100 & 10 & 1 \\ 400 & 20 & 1 \\ 1600 & 40 & 1 \end{vmatrix} = 1000\begin{vmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 16 & 4 & 1 \end{vmatrix} = 1000[-2+12-6] = -6000,$$

$$\Delta_1 = \begin{vmatrix} 8 & 10 & 1 \\ 16 & 20 & 1 \\ 22 & 40 & 1 \end{vmatrix} = 20\begin{vmatrix} 4 & 1 & 1 \\ 8 & 2 & 1 \\ 11 & 4 & 1 \end{vmatrix} = 20[-8+3+10] = 100,$$

$$\Delta_2 = \begin{vmatrix} 100 & 8 & 1 \\ 400 & 16 & 1 \\ 1600 & 22 & 1 \end{vmatrix} = 200\begin{vmatrix} 1 & 4 & 1 \\ 4 & 8 & 1 \\ 16 & 11 & 1 \end{vmatrix} = 200[-3+48-84] = -7800,$$

$$\Delta_3 = \begin{vmatrix} 100 & 10 & 8 \\ 400 & 20 & 16 \\ 1600 & 40 & 22 \end{vmatrix} = 2000\begin{vmatrix} 1 & 1 & 4 \\ 4 & 2 & 8 \\ 16 & 4 & 11 \end{vmatrix} = 2000[-10+84-64] = 20000.$$

By Cramer's rule, we get $a = \frac{\Delta_1}{\Delta} = -\frac{1}{60}$, $b = \frac{\Delta_2}{\Delta} = \frac{7800}{6000} = \frac{78}{60} = \frac{13}{10}$, $c = \frac{\Delta_3}{\Delta} = \frac{20000}{6000} = -\frac{20}{6} = -\frac{10}{3}$.

So, the equation of the path is $y = \frac{1}{60}x^2 + \frac{13}{10}x - \frac{10}{3}$.

When $x = 70$, we get $y = 6$. So, the ball went by 6 metres high over the boundary line and it is impossible for a fielder standing even just before the boundary line to jump and catch the ball. Hence the ball went for a super six and the Chennai Super Kings won the match.

5) Since $v(3) = 64$, $v(6) = 133$, and $v(9) = 208$, we get the following system of linear equations

$9a + 3b + c = 64$,

$36a + 6b + c = 133$,

$81a + 9b + c = 208$.

We solve the above system of linear equations by Gaussian elimination method.

Reducing the augmented matrix to an equivalent row-echelon form by using elementary row operations, we get

$[A \mid B] =$